Math 115 — Second Midterm

March 25, 2010

Name: _____

Instructor: _____ Section: _____

1. **Do not open this exam until you are told to do so.**
2. This exam has 9 pages including this cover. There are 8 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.
4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it. Include units in your answer where that is appropriate.
6. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
7. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
8. **Turn off all cell phones and pagers**, and remove all headphones.
9. **Use the techniques of calculus to solve the problems on this exam.**

Problem	Points	Score
1	12	
2	12	
3	10	
4	12	
5	13	
6	12	
7	16	
8	13	
Total	100	

1. [12 points]
For the following statements, select True if the statement is *ALWAYS* true, and select False otherwise. No explanations are required.

a. [2 points] Suppose that f is a function whose second derivative is both continuous and positive everywhere. Then

$$f(2+\Delta x) > f(2) + f'(2)\Delta x.$$

True False

b. [2 points] Suppose that g is a continuous function and g' is defined for all x. Then g'' is also defined for all x.

True False

c. [2 points] If a continuous function H has exactly one local maximum and two local minima, then there are exactly three distinct values of x such that $H'(x) = 0$.

True False

d. [2 points] Suppose that A and B are two continuous functions such that $A'(x) \leq B'(x)$ for all x. Then $A(x) \leq B(x)$ for all x.

True False

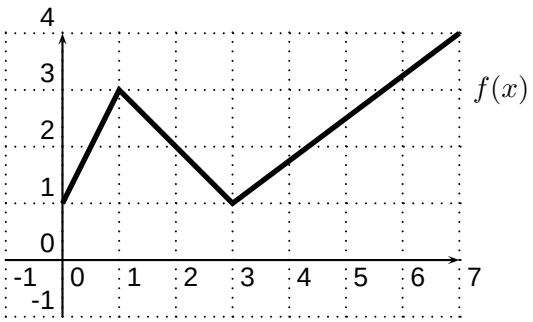
e. [2 points] Suppose $P(x)$ is a continuous function satisfying $P'(x) \geq 0$ whenever $x > 0$. Then $P(a) \leq P(b)$ whenever $0 < a < b$.

True False

f. [2 points] If the functions R and S are inverses of each other, then R' and S' are inverses of each other.

True False

2. [12 points]
Use the graph of the function f and the table of values for the function g to answer the questions below.



x	1	2	3	4	5	6
g(x)	0	4	0	-18	-56	-120
g'(x)	6	1	-10	-27	-50	-79
g"(x)	-2	-8	-14	-20	-26	-32

a. [6 points] Let $h(x) = \frac{g(x)}{f(2x+3)}$. Find $h'(1)$ or explain why it does not exist.

b. [6 points] Let $k(x) = g(g(x))$. Determine whether k is increasing or decreasing at $x = 2$.

3. [10 points]
Find the x- and y-coordinates of all local minima, local maxima, and inflection points of the function $f(x)$ defined below. Your answers may involve the positive constant B. You must clearly mark your answers and provide justification to receive credit.

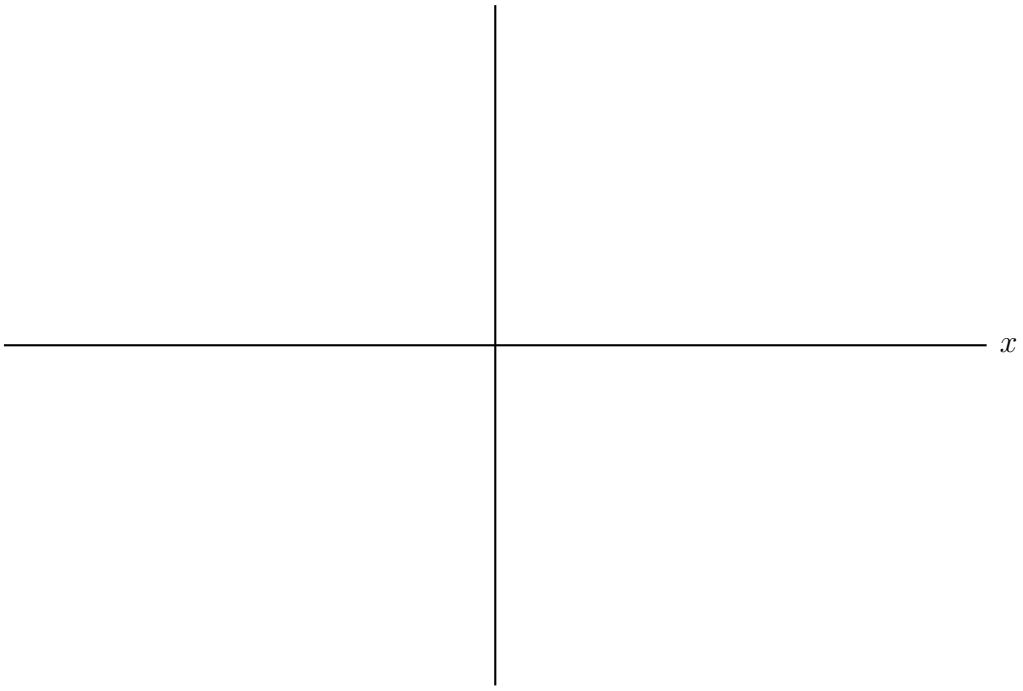
$$f(x) = e^{-18x^2+B}$$

4. [12 points]

The two parts below are independent. Be sure to label any relevant features of your graphs.

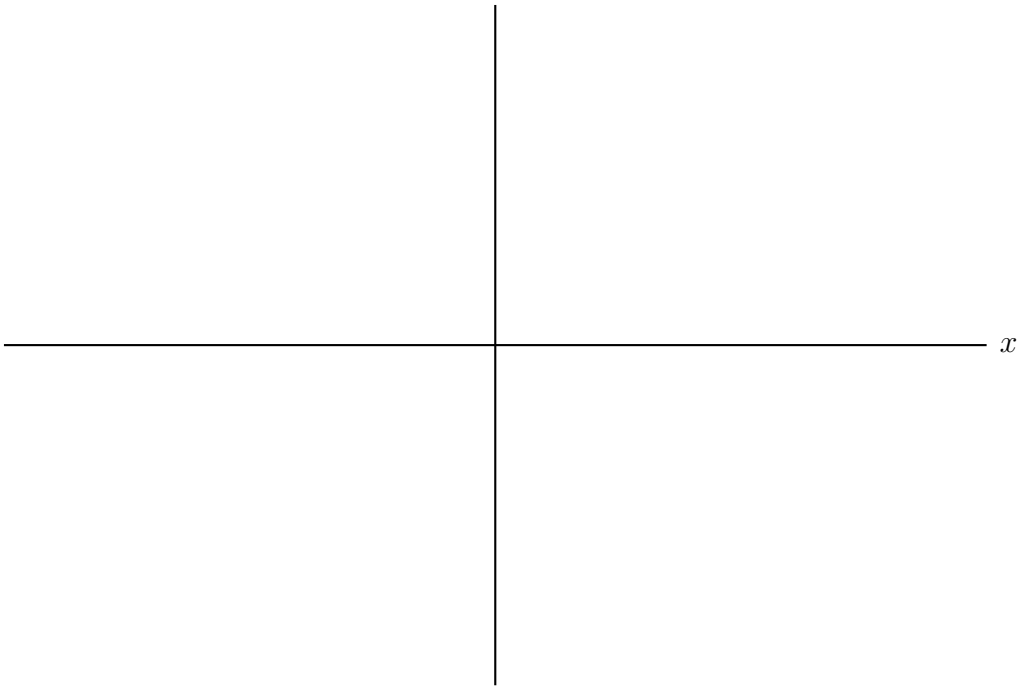
a. [6 points] Draw an example of a continuous function $f(x)$ such that

- f has a critical point at $x = -2$ and $f'(-2) \neq 0$, and
- f has a critical point at $x = 3$ and $f'(3) = 0$.



b. [6 points] Draw the *derivative* of a function $g(x)$ satisfying

- g is decreasing on the interval $(-\infty, 0)$, and
- $g''(x) > 0$ when $x > 0$.



5. [13 points] The equation below implicitly defines a hyperbola.

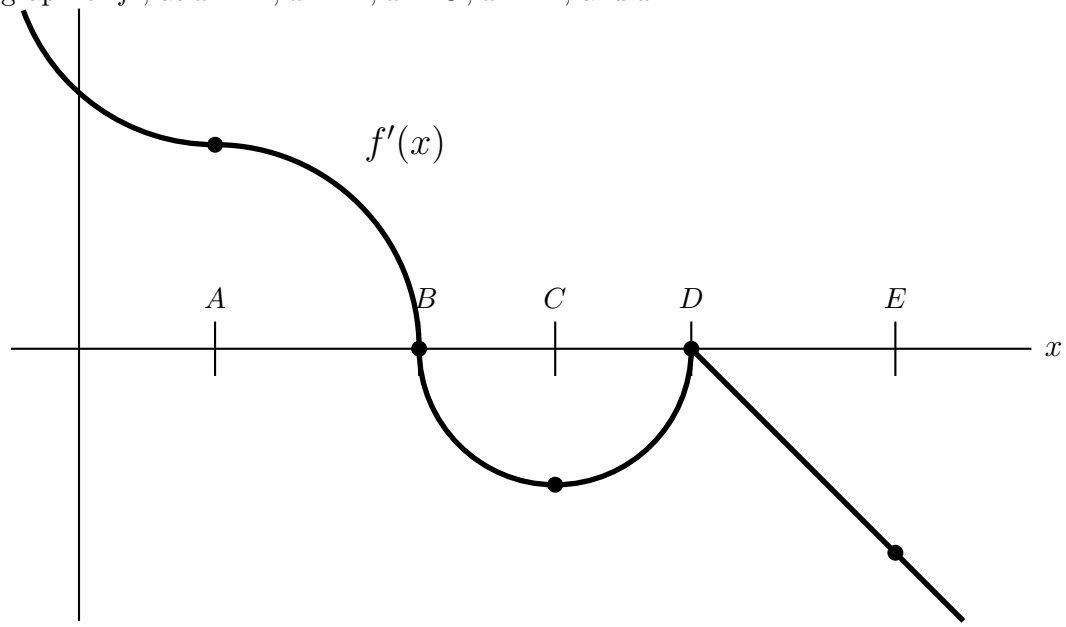
$$x^2 - y^2 = 2x + xy + y + 2.$$

a. [5 points] Find $\frac{dy}{dx}$.

b. [4 points] Consider the two points $(4, 2)$ and $(2, -1)$. Show that one of these points lies on the hyperbola defined above, and one does not.

c. [4 points] For the point in part (b) which lies on the hyperbola, find the equation of the line which is tangent to the hyperbola at this point.

6. [12 points] The *derivative* of a function f is graphed below. Five points are marked on the graph of f', at $x = A$, $x = B$, $x = C$, $x = D$, and $x = E$.



For each of the following, circle ALL answers which are correct. Each part has at least one correct answer. Pay careful attention to whether each question is asking about f, f', or f''.

a. [2 points] The function f' has a local minimum when _____.

$x = A$ $x = B$ $x = C$ $x = D$ $x = E$

b. [2 points] The function f is increasing when _____.

$x = A$ $x = B$ $x = C$ $x = D$ $x = E$

c. [2 points] The function f has a critical point when _____.

$x = A$ $x = B$ $x = C$ $x = D$ $x = E$

d. [2 points] The global maximum of f on the interval $A \leq x \leq E$ occurs when _____.

$x = A$ $x = B$ $x = C$ $x = D$ $x = E$

e. [2 points] The function f has an inflection point when _____.

$x = A$ $x = B$ $x = C$ $x = D$ $x = E$

f. [2 points] The function f'' is undefined when _____.

$x = A$ $x = B$ $x = C$ $x = D$ $x = E$

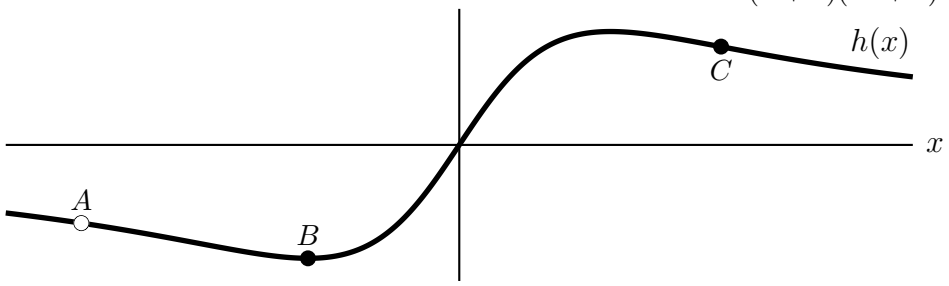
7. [16 points] Janet is an artist who produces and sells prints of her artwork. If Janet sells her prints for \$17 each, then she will sell 340 prints. Janet is considering whether she should change the price. She takes a survey and concludes that for each price increase of 75 cents, she will sell 10 fewer prints.

a. [4 points] Find a formula for Janet's revenue, $R(x)$, in terms of x, the number of 75 cent price increases.

b. [4 points] Janet plans to produce exactly the number of prints that her survey predicts she will sell. Her costs include \$2 per print, along with \$500 in fixed costs. Find a formula for $C(x)$, Janet's total costs, in terms of x, the number of 75 cent price increases.

c. [8 points] Use the methods of calculus to determine what price Janet should set for her prints if she wants to maximize her profit.

8. [13 points] Below, there is a graph of the function $h(x) = \dfrac{2x^2 + 10x}{(x+5)(x^2+4)}$.



a. [3 points] The point A is a hole in the graph of h. Find the x- and y-coordinates of A.

b. [5 points] The point B is a local minimum of h. Find the x- and y-coordinates of B.

c. [5 points] The point C is an inflection point of h. Find the x- and y-coordinates of C.