Math 115 — Second Midterm

March 22, 2011

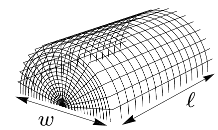
Name: ____________________

Instructor: ____________________ Section: __________

1. **Do not open this exam until you are told to do so.**
2. This exam has 8 pages including this cover. There are 7 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.
4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it. Include units in your answer where that is appropriate.
6. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
7. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
8. **Turn off all cell phones and pagers**, and remove all headphones.
9. Note that problems 5–7 will be graded giving very little partial credit.

Problem	Points	Score
1	15	
2	16	
3	12	
4	15	
5	15	
6	15	
7	12	
Total	100	

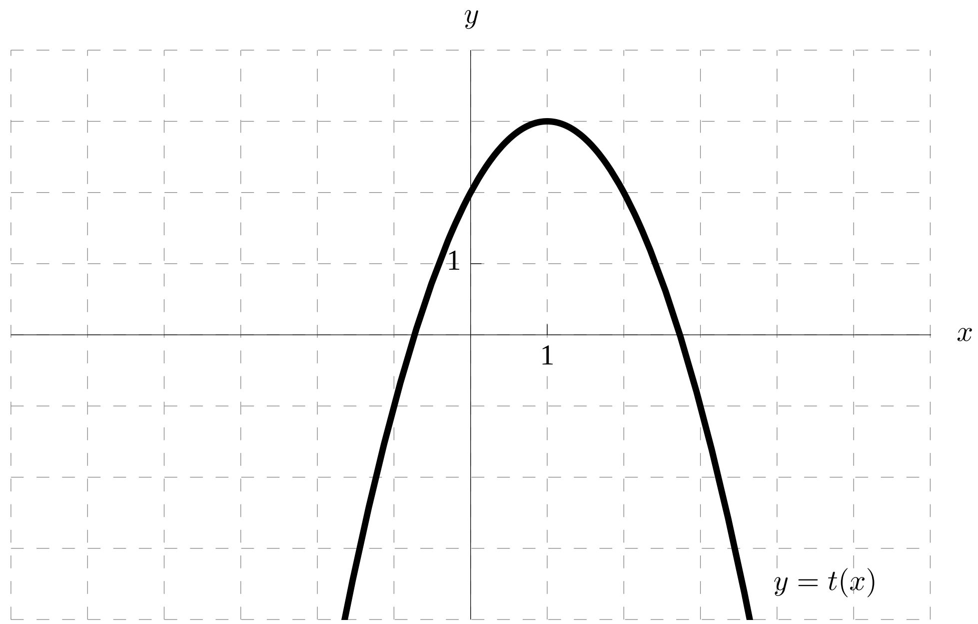
1. [15 points] A hoophouse is an unheated greenhouse used to grow certain types of vegetables during the harsh Michigan winter. A typical hoophouse has a semi-cylindrical roof with a semi-circular wall on each end (see figure to the right). The growing area of the hoophouse is the rectangle of length ℓ and width w (each measured in feet) which is covered by the hoophouse. The cost of the semi-circular walls is \$0.50 per square foot and the cost of the roof, which varies with the side length ℓ, is $\$1 + 0.001\ell$ per square foot.



a. [4 points] Write an equation for the cost of a hoophouse in terms of ℓ and w. *(Hint: The surface area of a cylinder of height ℓ and radius r, not including the circles on each end, is $A = 2\pi r\ell$.)*

b. [11 points] Find the dimensions of the least expensive hoophouse with 8000 square feet of growing area.

2. [16 points]
Graphed below is a function $t(x)$. Define $p(x) = x^2 t(x)$, $q(x) = t(\sin(x))$, $r(x) = \frac{t(x)}{3x+1}$, and $s(x) = t(t(x))$. For this problem, do not assume $t(x)$ is quadratic.



Carefully estimate the following quantities.

a. [4 points] $p'(-1)$

b. [4 points] $q'(0)$

c. [4 points] $r'(3)$

d. [4 points] $s'(0)$

3. [12 points] Representative values of the derivative of a function $f(x)$ are shown in the table below. Assume $f'(x)$ is a continuous function and that the values in the table are representative of the behavior of $f'(x)$.

x	0	0.5	1	1.5	2	2.5	3
$f'(x)$	1	0.3	0	-0.1	-0.15	-0.12	-0.10

a. [6 points] Estimate the location of the global maximum and minimum of $f(x)$ on the closed interval $[0, 3]$. Justify your answers based on the data in the table.

b. [6 points] Can you tell from these data if $f(x)$ has any inflection points? If so, estimate the location of any inflection points and indicate how you know their locations. If not, explain why not.

4. [15 points] A model for the amount of an antihistamine in the bloodstream after a patient takes a dose of the drug gives the amount, a, as a function of time, t, to be $a(t) = A(e^{-t} - e^{-kt})$. In this equation, A is a measure of the dose of antihistamine given to the patient, and k is a transfer rate between the gastrointestinal tract and the bloodstream. A and k are positive constants, and for pharmaceuticals like antihistamine, $k > 1$.

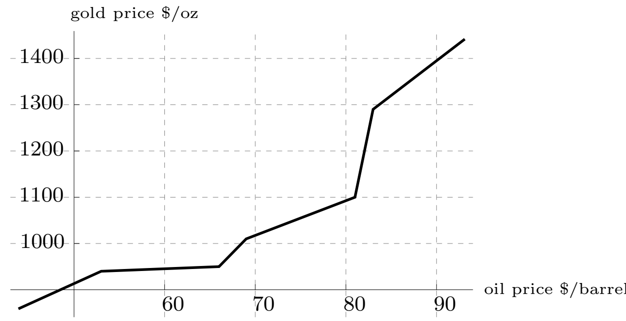
a. [5 points] Find the location $t = T_m$ of the non-zero critical point of $a(t)$.

b. [3 points] Explain why $t = T_m$ is a global maximum of $a(t)$ by referring to the expression for $a(t)$ or $a'(t)$.

c. [4 points] The function $a(t)$ has a single inflection point. Find the location $t = T_I$ of this inflection point. You do not need to prove that this is an inflection point.

d. [3 points] Using your expression for T_m from (a), find the rate at which T_m changes as k changes.

5. [15 points] The graph to the right shows a function $G(b)$ that approximates the price of an ounce of gold (in dollars) as a function of the cost of a barrel of oil for data between 2009 and 2011.[1]



a. [3 points] Estimate $G'(70)$.

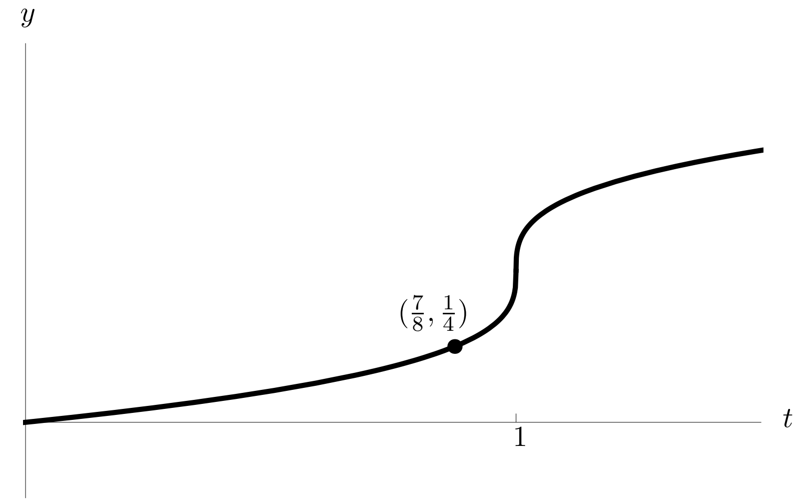
b. [5 points] Recall that G^{-1} is defined to be a function such that $G^{-1}(G(b)) = b$ (or such that $G(G^{-1}(y)) = y$, where y is the price of an ounce of gold). Derive, using the chain rule, a formula for $(G^{-1})'$ in terms of G'.

c. [4 points] Using parts (a) and (b), estimate $(G^{-1})'(G(70))$.

d. [3 points] Explain the practical meaning of your result in (c).

[1] Gold prices from <http://www.goldprice.org/>; oil from <http://en.wikipedia.org/wiki/Price_of_petroleum>.

6. [15 points] Given below is the graph of a function $h(t)$. Suppose $j(t)$ is the local linearization of $h(t)$ at $t = \frac{7}{8}$.



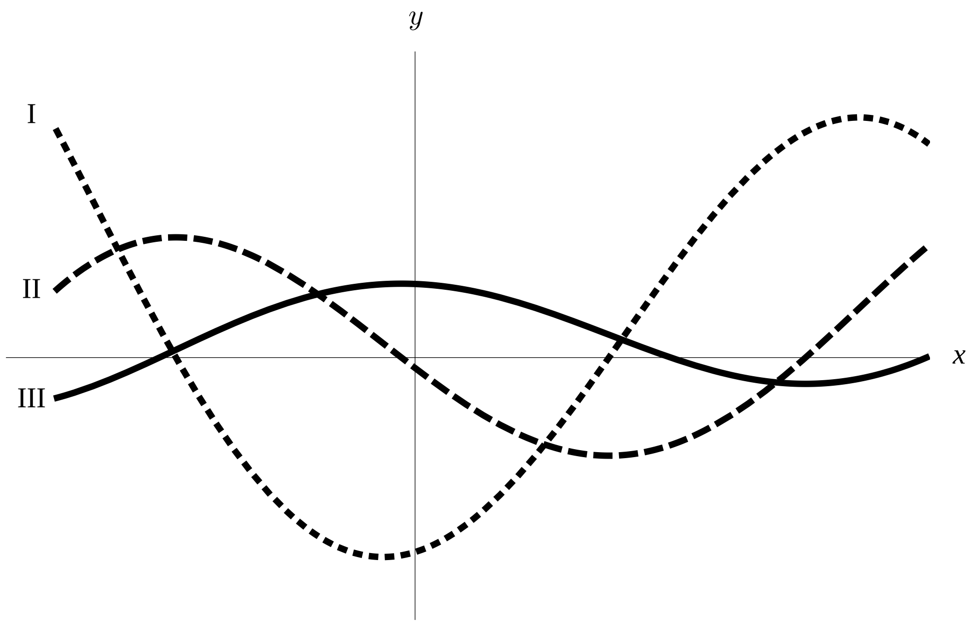
a. [5 points] Given that $h'(\frac{7}{8}) = \frac{2}{3}$, find an expression for $j(t)$.

b. [4 points] Use your answer from (a) to approximate $h(1)$.

c. [3 points] Is the approximation from (b) an over- or under-estimate? Explain.

d. [3 points] Using $j(t)$ to estimate values of $h(t)$, will the estimate be more accurate at $t = 1$ or at $t = \frac{3}{4}$? Explain.

7. [12 points] On the axes below are graphed f, f', and f''. Determine which is which, and justify your response with a brief explanation.



f: ______

f': ______

f'': ______