Math 115 — Second Midterm

March 20, 2012

Name: ______________________________

Instructor: ______________________ Section: __________

1. **Do not open this exam until you are told to do so.**
2. This exam has 10 pages including this cover. There are 8 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.
4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it. Include units in your answer where that is appropriate.
6. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
7. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
8. **Turn off all cell phones and pagers**, and remove all headphones.
9. You must use the methods learned in this course to solve all problems.

Problem	Points	Score
1	10	
2	10	
3	12	
4	12	
5	12	
6	16	
7	12	
8	16	
Total	100	

1. [10 points] For the following questions, circle "True" if the statement is **always** true, and otherwise circle "False". No justification is necessary.

a. [2 points] If $f(x)$ is a function with a local maximum at $x = c$, then $f'(c) = 0$.

True False

b. [2 points] If $g'(55) = g'(65) = 0$, then $g(x)$ is constant on the interval $55 \leq x \leq 65$.

True False

c. [2 points] The point $(\pi, 1)$ is on the curve defined by the implicit function $5\sin(xy) = \ln(y)$.

True False

d. [2 points] The function $A(x) = \dfrac{1}{R^2}\cos(Rx) + \dfrac{1}{2}x^2$ has an inflection point at $x = 0$, where R is a nonzero constant.

True False

e. [2 points] If $h'(x) < 0$ for all x in the interval $[2, 8]$, then the global maximum of $h(x)$ on that interval occurs at $x = 2$.

True False

2. [10 points] Lucy Lemon, the owner of a local lemonade stand, has observed that her lemonade sales are highly dependent on the temperature. Let $L(T)$ denote the number of cups of lemonade Lucy sells on a day whose average temperature is T° Fahrenheit. Below is a table of values for the **derivative**, $L'(T)$. Assume that between each pair of consecutive T-values in the table, L' is either strictly increasing or strictly decreasing.

T	50	55	60	65	70	75	80	85	90
$L'(T)$	-5	-4	-2	2	5	6	3	-2	-3

a. [4 points] Approximate the values of T at which $L(T)$ has a critical point, and classify each as a local maximum, local minimum, or neither. (No explanation is necessary.)

b. [6 points] Suppose on a day when the temperature is T° Fahrenheit, Lucy sells $R(T)$ dollars worth of lemonade. Assuming cups of lemonade sell for \$1.50 each, compute $R'(80)$, and write a sentence expressing the meaning of $R'(80)$ which would be understood by someone who knows no calculus.

3. [12 points] The following questions relate to the implicit function

$$y^2 + 4x = 4xy^2.$$

a. [4 points] Compute $\frac{dy}{dx}$.

b. [4 points] Find the equation for the tangent line to this curve at the point $(\frac{1}{3}, 2)$.

c. [4 points] Find the x- and y-coordinates of all points at which the tangent line to this curve is vertical.

4. [12 points] Consider the family of functions

$$f(x) = ax - e^{bx},$$

where a and b are positive constants.

a. [4 points] Any function $f(x)$ in this family has only one critical point. In terms of a and b, what are the x- and y-coordinates of that critical point?

b. [4 points] Is the critical point a local maximum or a local minimum? Justify your answer with either the first-derivative test or the second-derivative test.

c. [4 points] For which values of a and b will $f(x)$ have a critical point at $(1, 0)$?

5. [12 points]

a. [3 points] Find the local linearization $L(x)$ of the function

$$f(x) = (1 + x)^k$$

near $x = 0$, where k is a positive constant.

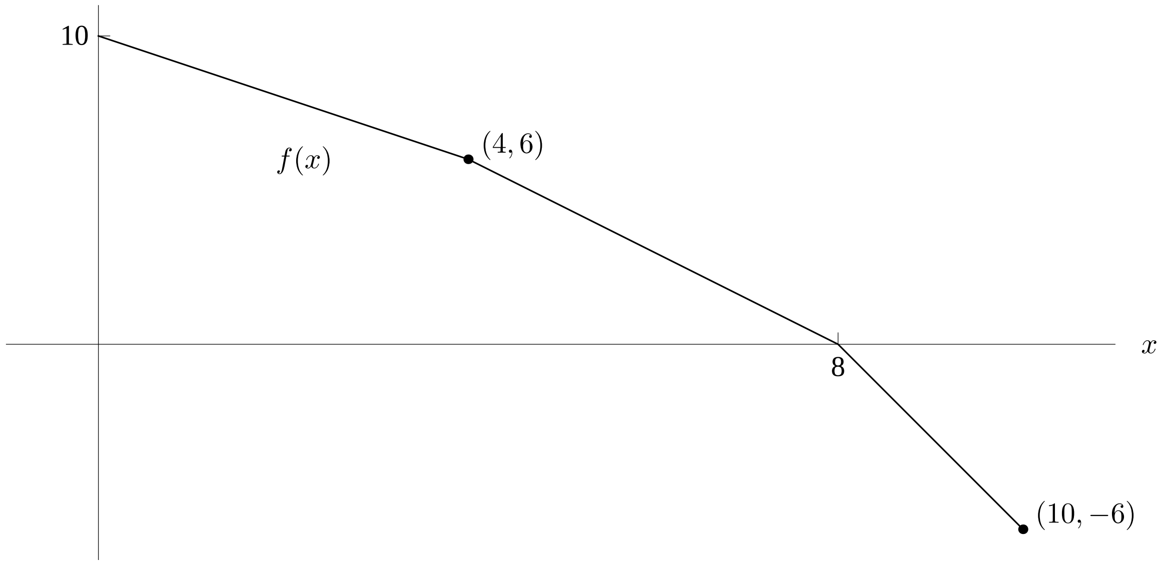
b. [3 points] For which values of k does this local linearization give underestimates of the actual value of $f(x)$? (Show your work.)

c. [2 points] Suppose you want to use $L(x)$ to find an approximation of the number $\sqrt{1.1}$. What number should k be, and what number should x be?

d. [2 points] Approximate $\sqrt{1.1}$ using $L(x)$.

e. [2 points] What is the error in the approximation from part (d)?

6. [16 points] Consider the piecewise linear function $f(x)$ graphed below:



For each function $g(x)$, find the value of $g'(3)$:

a. [4 points] $g(x) = \sin\left([f(x)]^3\right)$

b. [4 points] $g(x) = \dfrac{f(x^2)}{x}$

c. [4 points] $g(x) = \ln(f(x)) + f(2)$

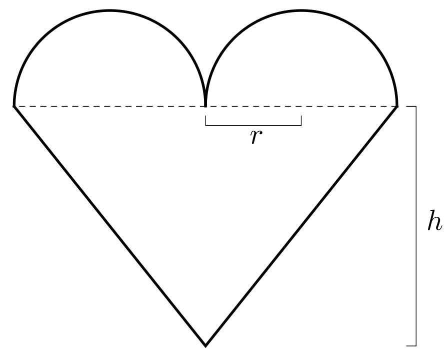
d. [4 points] $g(x) = f^{-1}(x)$

7. [12 points] For Valentine's Day, Jason decides to make a heart-shaped cookie for Sophie to try to win her over. Being mathematically-minded, the only kind of heart that Jason knows how to construct is composed of two half-circles of radius r and an isosceles triangle of height h, as shown below.

Jason happens to know that Sophie's love for him will be determined by the dimensions of the cookie she receives; if given a cookie as described above, her love L will be

$$L = hr^2,$$

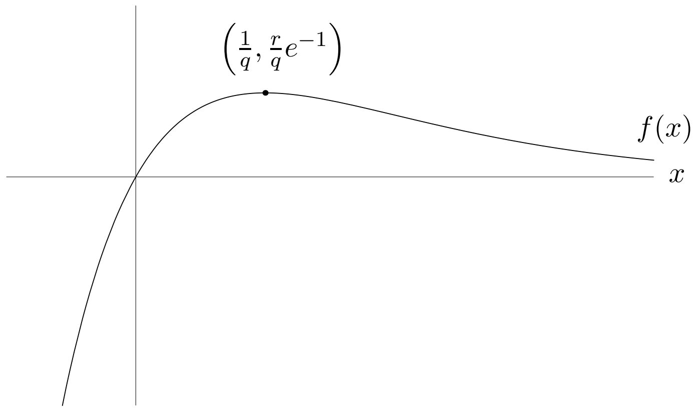
where r and h are measured in centimeters and L is measured in pitter-patters, a standard unit of affection. If Jason wants to make a cookie whose area is exactly 300cm^2, what should the dimensions be to maximize Sophie's love?



8. [16 points] Below is the graph of the function

$$f(x) = rxe^{-qx},$$

where r and q are constants. Assume that both r and q are greater than 1. The function $f(x)$ passes through the origin and has a local maximum at the point $P = \left(\frac{1}{q}, \frac{r}{q}e^{-1}\right)$, as shown in the graph.

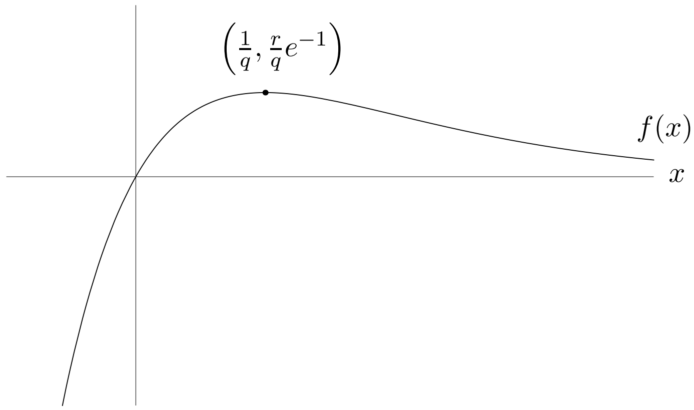


a. [4 points] Justify, using either the first-derivative test or second-derivative test, that the point P is a local maximum.

b. [2 points] What are the x-coordinates of the global maximum and minimum of $f(x)$ on the domain $[0, 1]$? (If $f(x)$ does not have a global maximum on this domain, say "no global maximum", and similarly if $f(x)$ does not have a global minimum.)

c. [2 points] What are the x-coordinates of the global maximum and minimum of $f(x)$ on the domain $(-\infty, \infty)$? (If $f(x)$ does not have a global maximum on this domain, say "no global maximum", and similarly if $f(x)$ does not have a global minimum.)

8. (continued) For your convenience, the graph of $f(x)$ is repeated below.



d. [4 points] Suppose that $g(x)$ is a function with $g'(x) = f(x)$. Find x-values of all local maxima and minima of $g(x)$. Justify that each maximum you find is a maximum and each minimum is a minimum.

e. [4 points] If $g(x)$ is as in part (d), for which x-values does $g(x)$ have inflection points? Show that these x-values are indeed inflection points.