Math 115 — Second Midterm

March 21, 2013

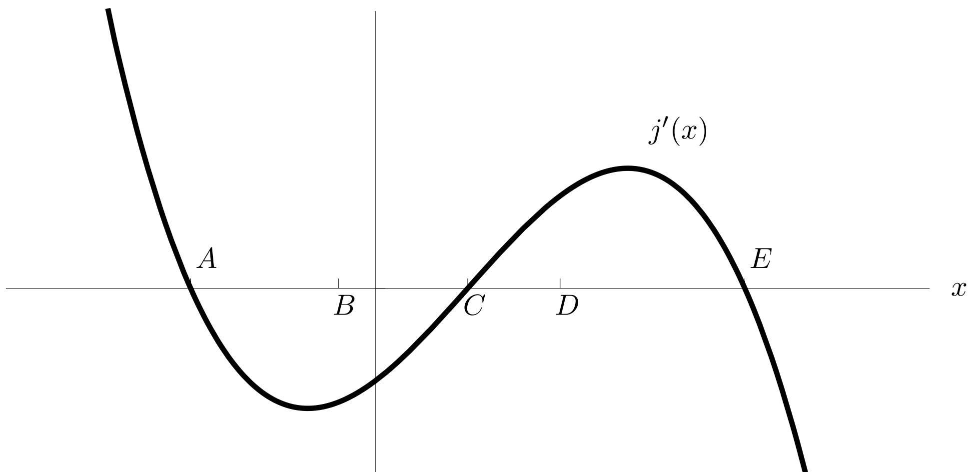
Name: ____________________

Instructor: ____________________ Section: __________

1. **Do not open this exam until you are told to do so.**
2. This exam has 10 pages including this cover. There are 9 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.
4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it. Include units in your answer where that is appropriate.
6. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
7. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
8. **Turn off all cell phones and pagers**, and remove all headphones.
9. You must use the methods learned in this course to solve all problems.

Problem	Points	Score
1	12	
2	10	
3	12	
4	11	
5	14	
6	9	
7	10	
8	12	
9	10	
Total	100	

1. [12 points] Consider the graph of $j'(x)$ given here. Note that this is not the graph of $j(x)$.



For each of (a)-(f) below, list **all** x-values labeled on the graph which satisfy the given statement in the blank provided. If the statement is not true at any of the labeled x-values, write "NP". You do not need to show your work. No partial credit will be given on each part of this problem.

(a) The function $j(x)$ has a local minimum at $x =$______________.

(b) The function $j(x)$ has a local maximum at $x =$______________.

(c) The function $j(x)$ is concave up at $x =$______________.

(d) The function $j(x)$ is concave down at $x =$______________.

(e) The function $j'(x)$ has a critical point at $x =$______________.

(f) The function $j''(x)$ is greatest at $x =$______________.

2. [10 points] The velocity, in cm per second, of an inchworm moving in a straight line can be modeled by a **sinusoidal** function, $v(t)$, where t is the number of seconds since the inchworm started moving. Suppose the inchworm reaches its maximum velocity of 0.1 cm/sec one second after it starts moving and its minimum velocity of 0 cm/sec two seconds after it starts moving.

a. [5 points] Find a formula for $v(t)$ which is consistent with the information above.

b. [2 points] Based on your answer from (a), find a formula for $a(t)$, the acceleration of the inchworm t seconds after it started moving.

c. [3 points] Based on your answers above, find a time when the inchworm's acceleration attains its largest positive value.

3. [12 points] Consider the family of linear functions

$$L(x) = ax - 3$$

and the family of functions

$$M(x) = a\sqrt{x}$$

where a is a nonzero constant number. Note that the number a is the same for both equations. Find a value of a for which $L(x)$ is *tangent* to the graph of $M(x)$. Also find the x and y coordinates of the point of tangency. Write your answers in the blanks provided.

$a =$ ____________________

$x =$ ____________________

$y =$ ____________________

4. [11 points]

a. [4 points] Find the tangent line approximation of the function

$$p(x) = 1 + x^k$$

near $x = 1$, where k is a positive constant.

b. [2 points] Suppose you want to use your tangent line from (a) to approximate the number $1 + \sqrt{0.95}$. What values of k and x would you plug in to your answer from (a)?

c. [2 points] Approximate $1 + \sqrt{0.95}$ using your tangent line from (a).

d. [3 points] Determine whether your approximation in (c) is an over- or underestimate. Be sure your reasoning is clear.

5. [14 points] Consider the family of functions

$$g(x) = \frac{ax^b}{\ln(x)}$$

where a and b are nonzero constants.

a. [4 points] Calculate $g'(x)$.

b. [6 points] Find values for a and b so that $g(e) = 1$ and $g'(e) = 0$.

c. [4 points] With the values of a and b you found in (b), is $x = e$ a local minimum of g, a local maximum of g or neither? Justify your answer.

6. [9 points] In each of the following problems, draw a *graph* of a function with *all* of the indicated properties. If there is no such function, then write "NO SUCH FUNCTION EXISTS". You do not need to write any explanations. No partial credit will be given on each part of this problem.

a. [3 points] A continuous function $f(x)$, whose domain is all real numbers, with the following four properties:
(i.) $f(x)$ attains a local minimum somewhere.
(ii.) $f(x)$ attains a local maximum somewhere.
(iii.) $f(x)$ does *not* attain a global minimum.
(iv.) $f(x)$ does *not* attain a global maximum.

b. [3 points] A continuous function $g(x)$, whose domain is the closed interval $[0, 1]$, with the following two properties:
(i.) $g(x)$ does *not* attain a global maximum on the interval $[0, 1]$
(ii.) $g(x)$ attains a global minimum on the interval $[0, 1]$.

c. [3 points] A differentiable function $j(x)$ with the following two properties:
(i.) The linear approximation to $j(x)$ at $x = 3$ gives an *overestimate* when used to approximate $j(2)$.
(ii.) The linear approximation to $j(x)$ at $x = 3$ gives an *underestimate* when used to approximate $j(4)$.

7. [10 points] For each real number k, there is a curve in the plane given by the equation

$$e^{y^2} = x^3 + k.$$

a. [4 points] Find $\frac{dy}{dx}$.

b. [3 points] Suppose that $k = 9$. There are two points on the curve where the tangent line is horizontal. Find the x and y coordinates of each one.

c. [3 points] Now suppose that $k = \frac{1}{2}$. How many points are there where the curve has a horizontal tangent line?

8. [12 points] In the following table, both f and g are differentiable functions of x. In addition, $g(x)$ is an invertible function. Write your answers in the blanks provided. You do not need to show your work.

x	2	3	4	5
$f(x)$	7	6	2	9
$f'(x)$	-2	1	3	2
$g(x)$	1	4	7	11
$g'(x)$	1	2	3	2

a. [3 points] If $h(x) = \frac{g(x)}{f(x)}$, find $h'(4)$.

$h'(4) =$ __________________

b. [3 points] If $k(x) = f(x)g(x)$, find $k'(2)$.

$k'(2) =$ __________________

c. [3 points] If $m(x) = g^{-1}(x)$, find $m'(4)$.

$m'(4) =$ __________________

d. [3 points] If $n(x) = f(g(x))$, find $n'(3)$.

$n'(3) =$ __________________

9. [10 points] The function $f(x)$ is twice-differentiable. Some values of f and f' are given in the following table. In addition, it is known that $f''(x)$ is positive.

x	0	1	2	3	4
$f(x)$	7	6	7	9	12
$f'(x)$	-2	$\frac{1}{2}$	1	2	4

No partial credit will be given on any part of this problem.

a. [4 points] **Circle** any statement which is true, and **draw a line through** any statement which is false.

(i.) For some value of x with $0 < x < 1$, f has a critical point.

(ii.) For some value of x with $1 < x < 2$, f has a critical point.

(iii.) For some value of x with $2 < x < 3$, f has a critical point.

(iv.) For some value of x with $3 < x < 4$, f has a critical point.

b. [3 points] If possible, find the global minimum value of $f(x)$ on the closed interval $[0, 4]$. (Give the y-coordinate, not the x-coordinate.) Do not give an approximation. If it is not possible to find it exactly, write "IT IS NOT POSSIBLE TO FIND IT EXACTLY."

c. [3 points] If possible, find the global maximum value of $f(x)$ on the closed interval $[0, 4]$. (Give the y-coordinate, not the x-coordinate.) Do not give an approximation. If it is not possible to find it exactly, write "IT IS NOT POSSIBLE TO FIND IT EXACTLY."