On my honor, as a student, I have neither given nor received unauthorized aid on this academic work. **Signed:** _____

Math 115 — Second Midterm

March 27, 2014

Name: _____

Instructor: _____ Section: _____

1. **Do not open this exam until you are told to do so.**
2. This exam has 11 pages including this cover. There are 11 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.
4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it.
6. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
7. For any graph or table that you use to find an answer, be sure to sketch the graph or write out the entries of the table. In either case, include an explanation of how you used the graph or table to find the answer.
8. Include units in your answer where that is appropriate.
9. **Turn off all cell phones, smartphones, and other electronic devices**, and remove all headphones.
10. You must use the methods learned in this course to solve all problems.

Problem	Points	Score
1	12	
2	9	
3	12	
4	8	
5	13	
6	10	
7	5	
8	6	
9	10	
10	10	
11	5	
Total	100	

1. [12 points] The table below gives several values of a differentiable function $f(x)$. Assume that both $f(x)$ and $f'(x)$ are invertible. Do not give approximations. If it is not possible to find the value exactly, write NOT POSSIBLE.

x	-3	-2	-1	0	1	2	3
$f(x)$	-8	-4	-1.2	0.5	1.4	1.8	2
$f'(x)$	5	3	2	1.2	0.5	0.3	0.1

a. [2 points] Let $g(x) = 3f(x) + 4$. Find $g'(1)$.

Answer: $g'(1) =$ _____

b. [2 points] Find $(f^{-1})'(2)$.

Answer: $(f^{-1})'(2) =$ _____

c. [2 points] Let $h(x) = f(e^x)$. Find $h'(\ln 2)$.

Answer: $h'(\ln 2) =$ _____

d. [2 points] Let $j(x) = e^{f(x)}$. Find $j'(-2)$.

Answer: $j'(-2) =$ _____

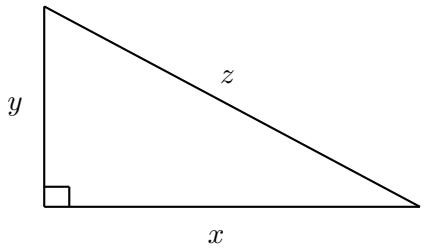
e. [2 points] Let $k(x) = f(x)f(x-2)$. Find $k'(1)$.

Answer: $k'(1) =$ _____

f. [2 points] Let $\ell(x) = \frac{f(x)}{f(x+3)}$. Find $\ell'(0)$.

Answer: $\ell'(0) =$ _____

2. [9 points] Consider a right triangle with legs of length x ft and y ft and hypotenuse of length z ft, as in the following picture:



a. [2 points] Suppose that the perimeter of the triangle is 8 ft. Let $A(x)$ give the area of the triangle, in ft^2, as a function of the side length x. In the context of this problem, what is the domain of $A(x)$? *Note that you do <u>not</u> need to find a formula for $A(x)$.*

Answer: _____

b. [7 points] Suppose instead that the perimeter of the triangle is allowed to vary, but the area of the triangle is fixed at 3 ft^2. Let $P(x)$ give the perimeter of the triangle, in ft, as a function of the side length x.

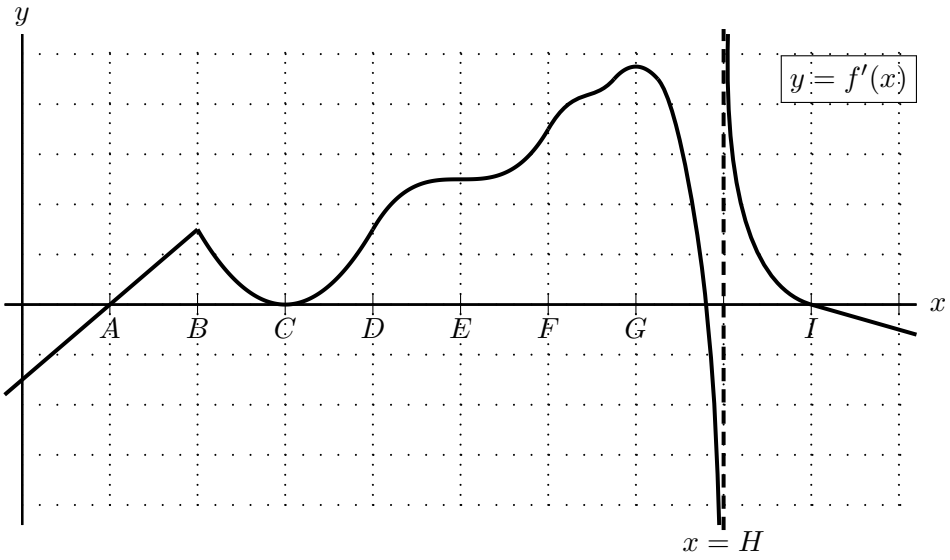
(i) In the context of this problem, what is the domain of $P(x)$?

Answer: _____

(ii) Find a formula for $P(x)$. The variables y and z should <u>not</u> appear in your answer. (This is the equation one would use to find the value(s) of x minimizing the perimeter. You should <u>not</u> do the optimization in this case.)

Answer: $P(x) =$ _____

3. [12 points] The graph of a portion of $y = f'(x)$, the derivative of $f(x)$ is shown below. Note that there is a sharp corner at $x = B$ and that $x = H$ is a vertical asymptote.
The function $f(x)$ is continuous with domain $(-\infty, \infty)$.



For each of the questions below, circle all of the available correct answers.
(Circle NONE if none of the available choices are correct.)

a. [2 points] At which of the following six values of x is the function $f(x)$ not differentiable?

B C E F H I NONE

b. [2 points] At which of the following six values of x does the function $f'(x)$ appear to be not differentiable?

A B C D E F NONE

c. [2 points] At which of the following nine values of x does $f(x)$ have a critical point?

A B C D E F G H I NONE

d. [2 points] At which of the following nine values of x does $f(x)$ have a local minimum?

A B C D E F G H I NONE

e. [2 points] At which of the following nine values of x is $f''(x) = 0$?

A B C D E F G H I NONE

f. [2 points] At which of the following nine values of x does $f(x)$ have an inflection point?

A B C D E F G H I NONE

4. [8 points] A ship's captain is standing on the deck while sailing through stormy seas. The rough waters toss the ship about, causing it to rise and fall in a sinusoidal pattern. Suppose that t seconds into the storm, the height of the captain, in feet above sea level, is given by the function

$$h(t) = 15\cos(kt) + c$$

where k and c are nonzero constants.

a. [3 points] Find a formula for $v(t)$, the vertical velocity of the captain, in feet per second, as a function of t. The constants k and c may appear in your answer.

Answer: $v(t) =$ _____

b. [2 points] Find a formula for $v'(t)$. The constants k and c may appear in your answer.

Answer: $v'(t) =$ _____

c. [3 points] What is the maximum vertical acceleration experienced by the captain? The constants k and c may appear in your answer. You do not need to justify your answer or show work. *Remember to include units.*

Answer: Max vertical acceleration: _____

5. [13 points] Suppose $f(x)$ is a function defined for all x whose derivative and second derivative are given by

$$f'(x) = \frac{(x+2)^2(x-3)}{(x+1)^{1/3}} \qquad \text{and} \qquad f''(x) = \frac{2(x+2)(x-1)(4x+3)}{3(x+1)^{4/3}}.$$

a. [2 points] Find the x-coordinates of all critical points of $f(x)$. If there are none, write NONE.

Answer: Critical point(s) at $x =$ _____

b. [6 points] Find the x-coordinates of all local extrema of $f(x)$.
If there are none of a particular type, write NONE.
Justify your answers, and be sure to show enough evidence to demonstrate that you have found all local extrema.

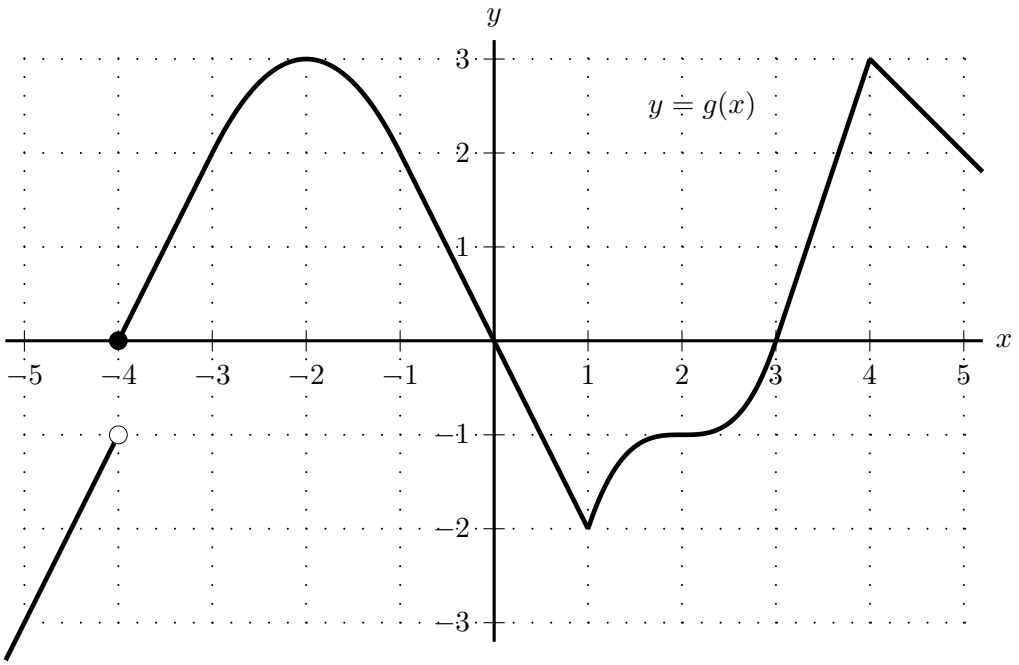
Answer: Local min(s) at $x =$ _____

Answer: Local max(es) at $x =$ _____

c. [5 points] Find the x-coordinates of all inflection points of $f(x)$. If there are none, write NONE.
Justify your answers, and be sure to show enough evidence to demonstrate that you have found all inflection points.

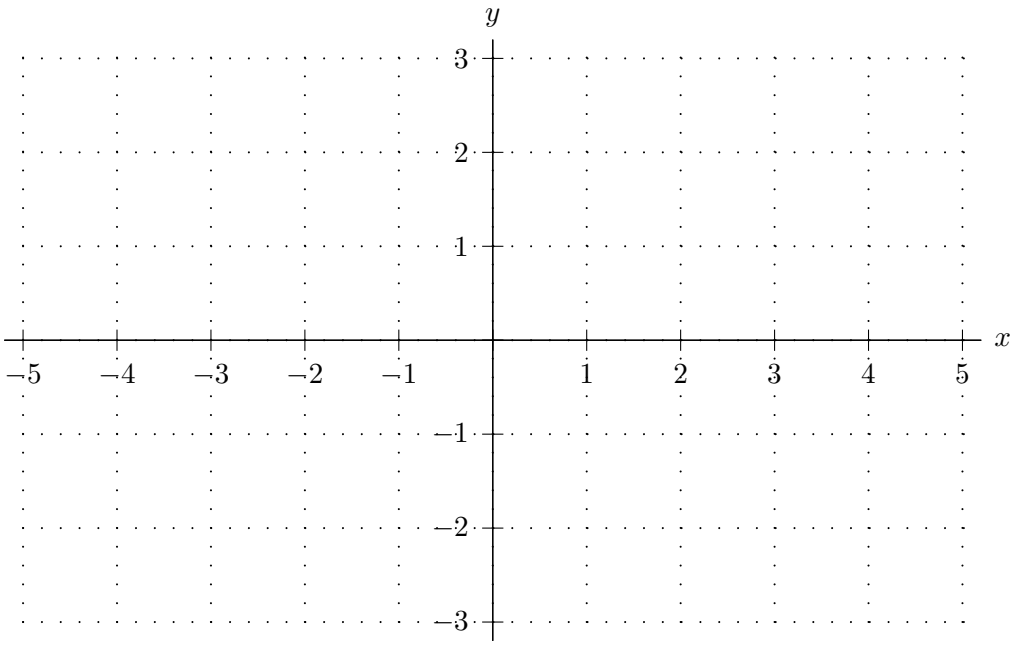
Answer: Inflection point(s) at $x =$ _____

6. [10 points] A portion of the graph of $y = g(x)$ is shown below.



On the axes below, sketch the graph of $y = g'(x)$.
Be sure that you pay close attention to each of the following:

- where g' is defined
- the value of $g'(x)$ near each of $x = -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$
- the sign of g'
- where g' is increasing/decreasing/constant



7. [5 points] Let

$$s(t) = \begin{cases} 5t^2 & \text{if } t \leq 3 \\ p + c(t-3) & \text{if } t > 3 \end{cases}$$

be a differentiable function, where p and c are constants.

a. [3 points] Find the values of p and c.

Answer: $p =$ _____ and $c =$ _____

b. [2 points] Is $s'(t)$ differentiable at $t = 3$?
To receive any credit on this question, you must justify your answer.

8. [6 points] Find a formula for $\frac{dy}{dx}$ for the implicit function $ax^2 + xy^2 + b \ln y = c$.
The constants a, b, and c may appear in your answer.

Answer: $\dfrac{dy}{dx} =$ _____

9. [10 points] After a long, cold winter, a ship's captain sails across Lake Michigan to Chicago. Upon arrival, the captain hosts a party on board to celebrate the arrival of spring. The party begins at exactly 6 pm and ends at exactly midnight. Let $N(t)$ be the noise level, in decibels, of the ship captain's party t hours after it begins. During the party, a formula for $N(t)$ is given by

$$N(t) = 0.5t^4 - 4t^3 + 7t^2 + 60.$$

a. [8 points] Find the <u>exact</u> values of t that minimize and maximize $N(t)$ on the interval $[0, 6]$. Use calculus to find your answers, and be sure to show enough evidence that the points you find are indeed global extrema.

(For each answer blank below, write NONE in the answer blank if appropriate.)

Answer: Global min(s) at <u>exactly</u> $t =$ _____

Answer: Global max(es) at <u>exactly</u> $t =$ _____

b. [2 points] How loud does the captain's party get? *Remember to include units.*

Answer: _____

10. [10 points] Let $f(x)$ be a function with $f(1) = 5$, $f'(1) = -2$, and $f''(1) = 3$.

a. [2 points] Use the local linearization of $f(x)$ at $x = 1$ to estimate $f(0.9)$.

Answer: $f(0.9) \approx$ _____

b. [2 points] Do you expect your estimate from Part (a) to be an overestimate or underestimate? To receive any credit on this question, you must justify your answer.

c. [2 points] Use the tangent line approximation of $f'(x)$ near $x = 1$ to estimate $f'(1.1)$.

Answer: $f'(1.1) \approx$ _____

d. [4 points] Suppose that the tangent line approximation of $f(x)$ near $x = 8$ estimates $f(8.05)$ to be 3.75 and $f(8.1)$ to be 3.6. Find $f(8)$ and $f'(8)$.

Answer: $f(8) =$ _____ and $f'(8) =$ _____

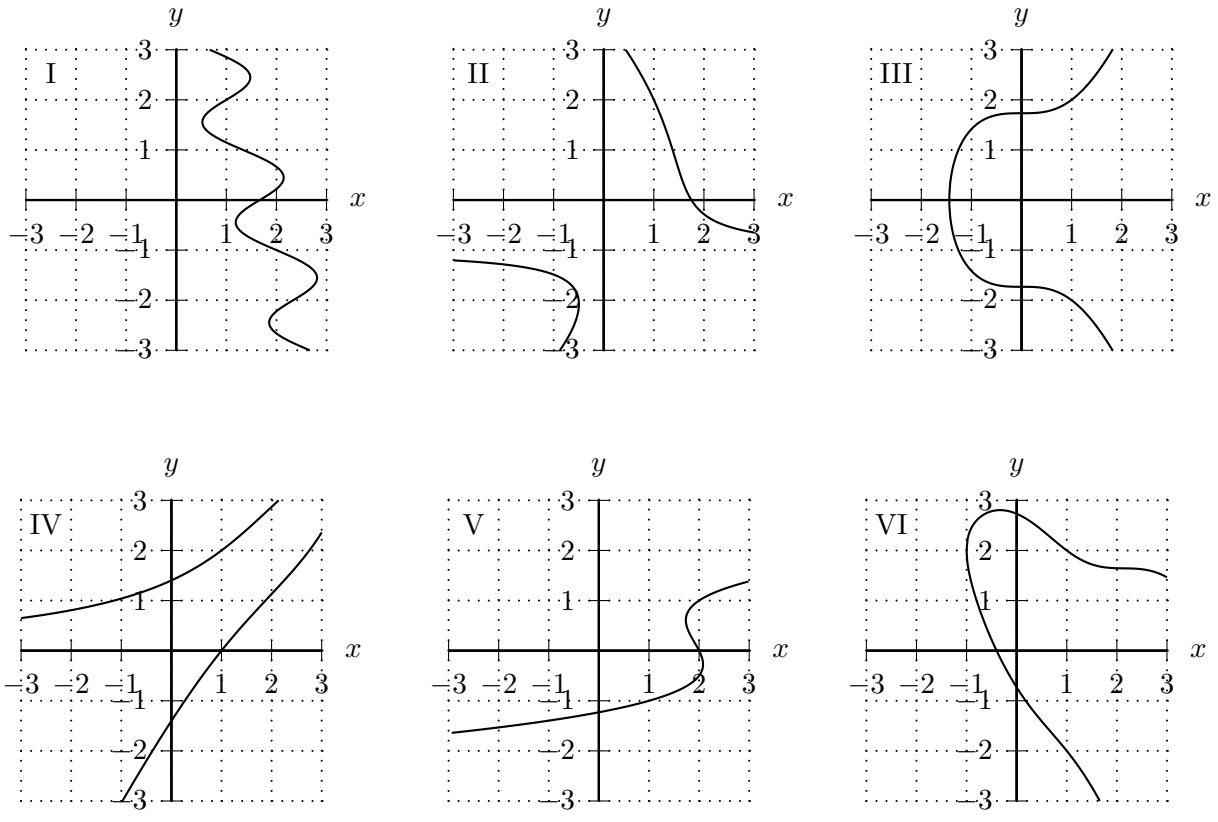
11. [5 points] A curve $\mathcal{C}$ gives y as an implicit function of x. The curve $\mathcal{C}$ passes through the point $(1, 2)$ and satisfies

$$\frac{dy}{dx} = \frac{y^2 - 2xy + 4y - 5}{4(y - x)}.$$

a. [1 point] One of the values below is the slope of the curve $\mathcal{C}$ at the point $(1, 2)$. Circle that one value.

Answer: The slope at $(1, 2)$ is $\quad \frac{1}{4} \quad \frac{1}{3} \quad \frac{1}{2} \quad \frac{5}{8} \quad \frac{2}{3} \quad \frac{3}{4} \quad \frac{4}{5}$

b. [4 points] One of the following graphs is the graph of the curve $\mathcal{C}$.
Which of the graphs I-VI is it? To receive any credit on this question, you must circle your answer next to the word "Answer" below.



Remember: To receive any credit on this question, you must circle your answer next to the word "Answer" below.

Answer: I II III IV V VI