Math 115 — Final Exam

April 21, 2016

EXAM SOLUTIONS

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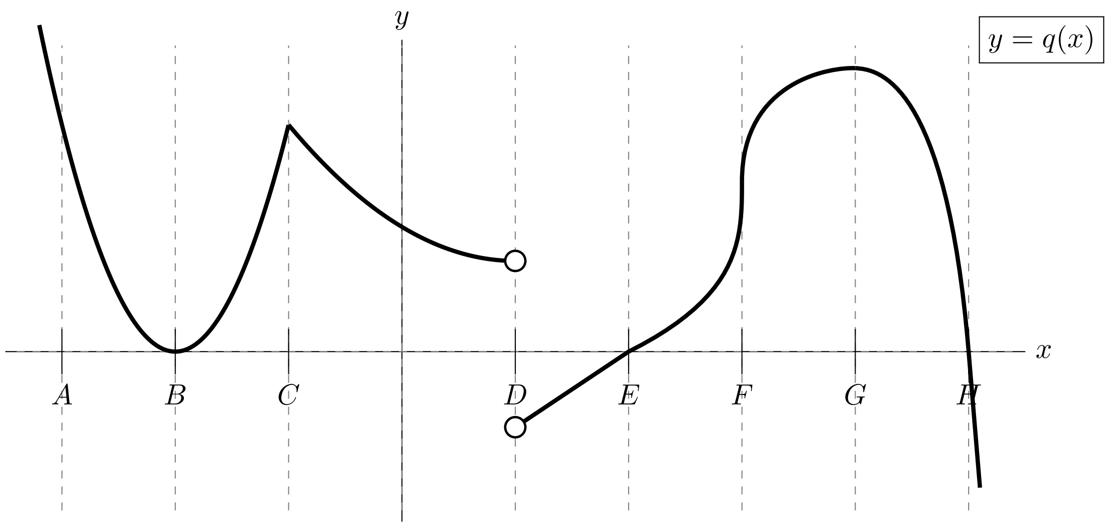
1. **Do not open this exam until you are told to do so.**
2. This exam has 11 pages including this cover. There are 10 problems.
   Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
3. Do not separate the pages of this exam. If they do become separated, write your initials (not name) on every page and point this out to your instructor when you hand in the exam.
4. Note that the back of every page of the exam is blank, and, if needed, you may use this space for scratchwork. Clearly identify any of this work that you would like to have graded.
5. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
6. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it.
7. The use of any networked device while working on this exam is not permitted.
8. You may use any calculator that does not have an internet or data connection except a TI-92 (or other calculator with a "qwerty" keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
9. For any graph or table that you use to find an answer, be sure to sketch the graph or write out the entries of the table. In either case, include an explanation of how you used the graph or table to find the answer.
10. Include units in your answer where that is appropriate.
11. **Turn off all cell phones, smartphones, and other electronic devices**, and remove all headphones, earbuds, and smartwatches.
12. You must use the methods learned in this course to solve all problems.

Problem	Points	Score
1	12	
2	8	
3	8	
4	15	
5	8	

Problem	Points	Score
6	10	
7	9	
8	7	
9	8	
10	10	
Post Test	5	
Total	100	

1. [12 points] A portion of the graph of a function $q(x)$ is shown below. Note that

- the graph of $y = q(x)$ has a sharp corner at $x = C$,
- the x−intercepts of the graph of $y = q(x)$ are at $x = B$, $x = E$, and $x = H$, and
- the tangent line to the graph of $y = q(x)$ at $x = F$ is vertical.



Let $Q(x)$ be an antiderivative of $q(x)$ that is defined and continuous on the interval $A \le x \le H$.

For each of the questions below, circle ALL of the available correct answers.
(Circle NONE if none of the available choices are correct.)

a. [2 points] At which of the following six values of x is $q(x)$ not differentiable?

A B $\boxed{C}$ $\boxed{F}$ G H NONE

b. [2 points] At which of the following eight values of x does $q(x)$ have a local maximum?

A B $\boxed{C}$ D E F $\boxed{G}$ H NONE

c. [2 points] At which of the following eight values of x does $Q(x)$ have a critical point?

A $\boxed{B}$ C $\boxed{D}$ $\boxed{E}$ F G $\boxed{H}$ NONE

d. [2 points] At which of the following eight values of x does $Q(x)$ have a local maximum?

A B C $\boxed{D}$ E F G $\boxed{H}$ NONE

e. [2 points] At which of the following eight values of x does $Q(x)$ have an inflection point?

A $\boxed{B}$ $\boxed{C}$ $\boxed{D}$ E F $\boxed{G}$ H NONE

f. [2 points] At which of the following seven values of x is $q'(x)$ (the derivative of q) a negative number?

$\boxed{A}$ B C E F G $\boxed{H}$ NONE

2. [8 points] Due to an accident, an oil pipeline is leaking.
Let $p(t)$ be the rate (in gallons/hour) at which the pipeline leaks oil t hours after the accident.
Assume that $p(t)$ is a strictly decreasing, differentiable function for $0 \leq t \leq 24$.
Engineers make the following measurements of $p(t)$.

t	0	6	12	18	24
$p(t)$	97	86	79	61	49

a. [2 points] Use the data provided, to estimate $p'(15)$. *Remember to show your work clearly.*

Solution:

$$p'(15) \approx \frac{p(18) - p(12)}{18 - 12} = \frac{61 - 79}{18 - 12} = -3$$

Answer: $p'(15) \approx$ -3

b. [3 points] Based on the data provided, write the right Riemann sum that best approximates the total amount of oil (in gallons) that leaked from the pipeline in the first 24 hours after the accident. *Be sure to carefully write out all of the terms in the sum.*

Solution: We want to estimate $\int_0^{24} p(t)\,dt$. Based on the limited data provided, the best we can do is to use 4 equal subintervals ($n = 4$, $\Delta t = 6$). The resulting approximation of the total number of gallons of oil that leaked from the pipeline in the first 24 hours after the accident is then

$$\int_0^{24} p(t)\,dt \approx p(6)\cdot 6 + p(12)\cdot 6 + p(18)\cdot 6 + p(24)\cdot 6 = 86(6) + 79(6) + 61(6) + 49(6) = 1650.$$

c. [1 point] Indicate whether the right sum above is an overestimate or an underestimate for the total amount of oil leaked. If there is not enough information to make this determination, circle “not enough information”. You do not need to explain your answer.

Answer: The right sum is an (circle one):

overestimate $\boxed{\text{underestimate}}$ not enough information

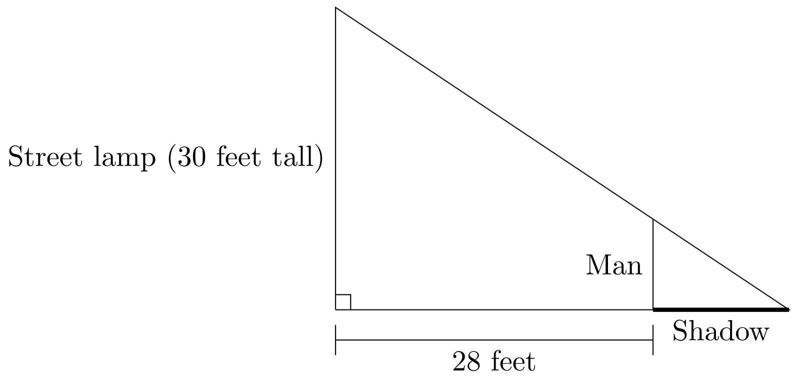
d. [2 points] Suppose that engineers measured $p(t)$ every 30 minutes, starting when the pipeline started leaking, for 24 hours. What would the difference (in gallons) be between the resulting best left sum and right sum estimates for the total amount of oil leaked in those first 24 hours?

Solution: The difference between the left and right Riemann sums for equal subintervals with $\Delta t = 0.5$ is

$$0.5\,(p(0) - p(24)) = 0.5(97 - 49) = 24.$$

Answer: $0.5(97 - 49) = 24$ **gallons**

3. [8 points] A man, who is 28 feet away from a 30 foot tall street lamp, is sinking into quicksand. (*See diagram below.*) At the moment when 6 feet of him are above the ground, his height above the ground is shrinking at a rate of 2 feet/second.



Throughout this problem, remember to show your work clearly, and include units in your answers.

a. [3 points] How long will the man's shadow (shown in bold in the diagram above) be at the moment when 6 feet of him are above the ground?

Solution: Let s be the length of the shadow. Noticing that the larger and smaller triangles in the picture are similar triangles, we have

$$\begin{aligned} \frac{30}{28+s} &= \frac{6}{s} \\ 30s &= 168 + 6s \\ 24s &= 168 \\ s &= 7. \end{aligned}$$

So the length of the shadow is 7 feet at that moment.

Answer: **7 feet**

b. [5 points] At what rate is the length of the man's shadow changing at the moment 6 feet of him are above the ground? Is his shadow growing or shrinking at that moment?

Solution: Let h be the height of the man above the ground, and let s be the length of his shadow. Using similar triangles as above, we have $\frac{30}{28+s} = \frac{h}{s}$ so $30s = 28h + hs$.

Taking derivatives with respect to time t, we find $30\frac{ds}{dt} = 28\frac{dh}{dt} + h\frac{ds}{dt} + s\frac{dh}{dt}$.

So at the moment when $h = 6$, we have

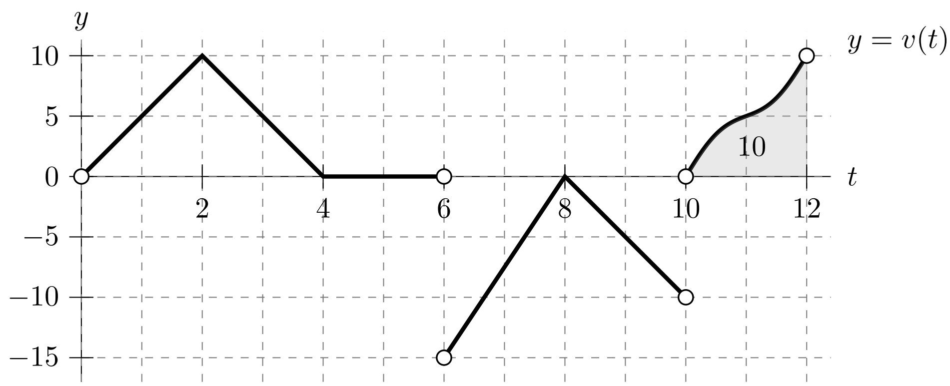
$$\begin{aligned} 30\left.\frac{ds}{dt}\right|_{h=6} &= 28(-2) + 6\left.\frac{ds}{dt}\right|_{h=6} + 7(-2) \\ 24\left.\frac{ds}{dt}\right|_{h=6} &= -70 \\ \left.\frac{ds}{dt}\right|_{h=6} &= \frac{-70}{24} = -\frac{35}{12} \approx -2.917 \end{aligned}$$

So at that moment, the shadow is shrinking at a rate of about 2.917 feet/second.

Answer: The man's shadow is (circle one) GROWING $\boxed{\text{SHRINKING}}$

at a rate of $\frac{35}{12}$ **(about 2.917) feet/second** .

4. [15 points] Elana goes on an amusement park ride that moves straight up and down. Let $v(t)$ model Elana's velocity (in meters/second) t seconds after the ride begins (where $v(t)$ is positive when the ride is moving upwards, and negative when the ride is moving downwards). A graph of $v(t)$ for $0 < t < 12$ is shown below. Assume that $v(t)$ is piecewise linear for $0 < t < 6$ and $6 < t < 10$, and that the area of the shaded region is 10, as indicated on the graph.



a. [4 points] Write an integral that gives Elana's average velocity, in meters/second, from 2 seconds into the ride until 4 seconds into the ride. Then compute the exact value of this integral.

Answer: $\frac{1}{4-2}\int_2^4 v(t)dt$ = 5

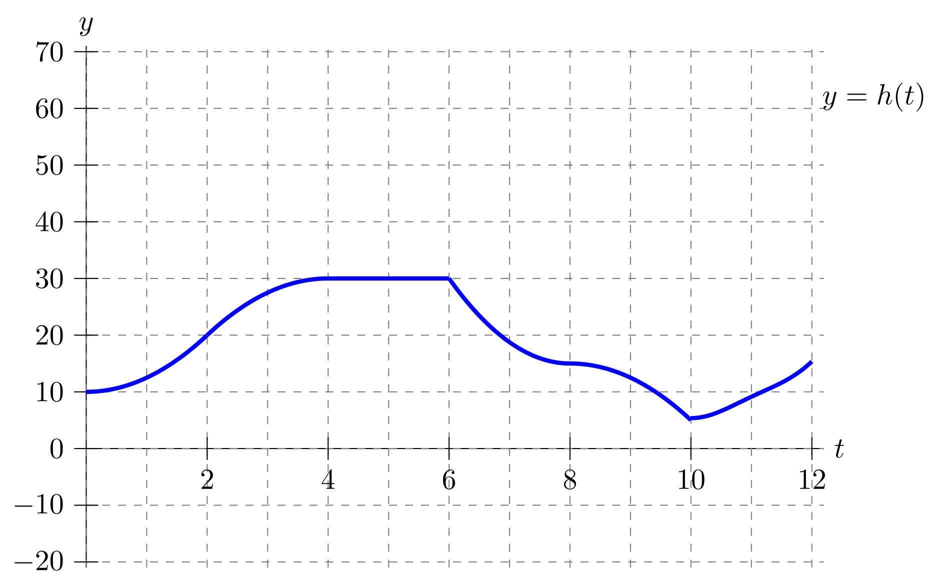
Let $h(t)$ be Elana's height (in meters) above the ground t seconds after the ride begins. Assume that h is continuous, and suppose Elana is at a height of 10 meters above the ground when the ride begins.

b. [6 points] Fill in the exact values of $h(t)$ in the table below.

t	0	2	4	6	8	10	12
$h(t)$	10	20	30	30	15	5	15

c. [5 points] Using your work from part **b.**, sketch a detailed graph of $h(t)$ for $0 < t < 12$. In your sketch, be sure that you pay close attention to each of the following:

- where h is increasing, decreasing, or constant
- the values of $h(t)$ you found in part **b.** above
- where h is/is not differentiable
- the concavity of the graph of $y = h(t)$



5. [8 points] Reggie is starting a fruit punch company. He has determined that the total cost, in dollars, for him to produce q gallons of fruit punch can be modeled by

$$C(q) = 100 + q + 25e^{q/100}.$$

Reggie can sell up to 100 gallons to Chris at a price of \$4 per gallon, and he can sell the rest to Alice at a price of \$3 per gallon. Assume that Reggie sells all of the fruit punch that he produces.
Note: Assume that the quantities of fruit punch produced and sold do not have to be whole numbers of gallons. (For example, Reggie could produce exactly $50\sqrt{2}$ gallons of fruit punch and sell all of these to Chris, who would pay a total of $200\sqrt{2}$ dollars for them.)

a. [4 points] For what quantities of fruit punch sold would Reggie's marginal revenue equal his marginal cost?

Solution: Reggie's marginal cost is $MC = C'(q) = 1 + \frac{1}{4}e^{q/100}$
and his marginal revenue is $MR = \begin{cases} 4 & \text{if } 0 < q < 100 \\ 3 & \text{if } 100 < q. \end{cases}$
So we solve $MR = MC$ separately for the two intervals $0 < q < 100$ and $q > 100$.

For $0 < q < 100$:

$$\begin{aligned} 1 + \frac{1}{4}e^{q/100} &= 4 \\ \frac{1}{4}e^{q/100} &= 3 \\ e^{q/100} &= 12 \\ q = 100\ln(12) &\approx 248.49. \end{aligned}$$

So marginal cost does not equal marginal revenue anywhere on the interval $0 < q < 100$ (because $100\ln(12) > 100$).

For $q > 100$:

$$\begin{aligned} 1 + \frac{1}{4}e^{q/100} &= 3 \\ \frac{1}{4}e^{q/100} &= 2 \\ e^{q/100} &= 8 \\ q = 100\ln(8) &\approx 207.94 \end{aligned}$$

Hence, marginal revenue equals marginal cost at $q = 100\ln(8)$.

Answer: $100\ln(8) \approx 207.94$ **gallons**

b. [4 points] Assuming that Reggie can produce at most 200 gallons of fruit punch, how much fruit punch should he produce in order to maximize his profit, and what would that maximum profit be? *You must use calculus to find and justify your answer. Be sure to provide enough evidence to justify your answer fully.*

Solution: First, we find all critical points of the profit function $\pi(q)$ in the interval $0 \le q \le 200$. In part **a.**, we found that $\pi'(q) = 0$ only at $q \approx 207.94$, which is not in the interval $[0, 200]$. The other critical points of $\pi(q)$ occur where $\pi'(q)$ is not defined, namely, at $q = 100$.
Note that Reggie's revenue is a continuous function of q. So $\pi(q)$ is continuous on the interval $[0, 200]$ and we can apply the Extreme Value Theorem. It therefore suffices to compare the value of $\pi(q)$ at the endpoints ($q = 0$ and $q = 200$) and at the critical point ($q = 100$):

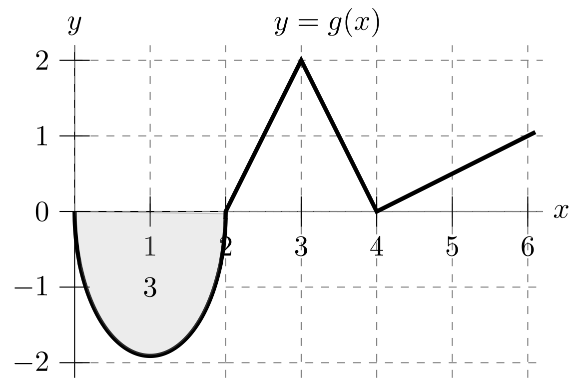
$$\begin{aligned} \pi(0) &= 0 - (100 + 0 + 25e^0) = -125 \\ \pi(100) &= 4(100) - (100 + 100 + 25e^1) \approx 132.04 \\ \pi(200) &= 4(100) + 3(100) - (100 + 200 + 25e^2) \approx 215.27 \end{aligned}$$

Hence, Reggie should produce 200 gallons of fruit punch for a profit of about \$215.27.

Answer: gallons of fruit punch: 200 and max profit: \$215.27

6. [10 points]

A portion of the graph of a continuous function $g(x)$ is shown on the right.
Assume that the area of the shaded region is 3 (as indicated on the graph), and note that $g(x)$ is piecewise linear for $2 < x < 6$.



For each of parts **a.-e.** below, find the value of the given quantity. If there is not enough information provided to find the value, write NOT ENOUGH INFO. If the value does not exist, write DOES NOT EXIST.

Remember to show your work.

a. [2 points] Find $\int_0^6 g(x)\,dx$.

Solution: $\int_0^6 g(x)\,dx = -3 + 2 + 1 = 0.$

Answer: 0

b. [2 points] Find $\int_0^2 (5 - 4g(x))\,dx$.

Solution: $\int_0^2 (5-4g(x))\,dx = \int_0^2 5\,dx - 4\int_0^2 g(x)\,dx = 5(2) - 4(-3) = 22.$

Answer: 22

c. [2 points] Suppose $C(x) = \ln(g(x))$. Find $C'(2.5)$.

Solution: $C'(x) = \dfrac{g'(x)}{g(x)}$, so $C'(2.5) = \dfrac{g'(2.5)}{g(2.5)} = \dfrac{2}{1} = 2.$

Answer: 2

d. [2 points] Find the average value of $g(x)$ on the interval $0 \le x \le 4$.

Solution: $\dfrac{1}{4-0}\int_0^4 g(x)\,dx = \dfrac{1}{4}(-3+2) = -\dfrac{1}{4}.$

Answer: $-\dfrac{1}{4}$

e. [2 points] Find $\int_2^4 (g(x+2) - g(x-2))\,dx$.

Solution:

$$\begin{aligned}\int_2^4 (g(x+2) - g(x-2))\,dx &= \int_2^4 g(x+2)\,dx - \int_2^4 g(x-2)\,dx \\ &= \int_4^6 g(x)\,dx - \int_0^2 g(x)\,dx \\ &= 1 - (-3) = 4\end{aligned}$$

Answer: 4

7. [9 points] Consider the family of functions given by $f(x) = e^{x^2+Ax+B}$ for constants A and B.

a. [6 points] Find and classify all local extrema of $f(x) = e^{x^2+Ax+B}$. Use calculus to find and justify your answers, and be sure to show enough evidence to demonstrate that you have found all local extrema. For each answer blank, write NONE if appropriate. Your answers may depend on A and/or B.

Solution: First, we find the critical points of $f(x)$. Notice that $f(x)$ is differentiable, so $f(x)$ only has critical points where $f'(x) = 0$. Since

$$f'(x) = (2x + A)e^{x^2+Ax+B},$$

the only critical point of $f(x)$ occurs where $2x+A = 0$, i.e., at $x = -\dfrac{A}{2}$. We test whether this critical point is a local maximum, a local minimum, or neither.
Applying the First Derivative Test:

- For $x < -\dfrac{A}{2}$: $2x + A < 0$ and $e^{x^2+Ax+B} > 0$, so $f'(x) < 0$.
- For $x > -\dfrac{A}{2}$: $2x + A > 0$ and $e^{x^2+Ax+B} > 0$, so $f'(x) > 0$.

Hence, $f'(x)$ changes from negative to positive at $x = -\dfrac{A}{2}$, so $f(x)$ has a local minimum at $x = -\dfrac{A}{2}$ (and no local maxima).
(Note that we could instead apply the Second Derivative Test: $f''(x) = (2x + A)(2x + A)(e^{x^2+Ax+B} + 2(e^{x^2+Ax+B}) = \left((2x + A)^2 + 2\right) e^{x^2+Ax+B}$ which is always positive (since both factors are always positive). So in particular $f''(-A/2) > 0$ so $f(x)$ has a local minimum at $x = -A/2$.

Answer: Local min(s) at $x =$ $-\dfrac{A}{2}$

Answer: Local max(es) at $x =$ NONE

b. [3 points] Find exact values of the constants A and B so that the point $(3, 1)$ is a critical point of $f(x) = e^{x^2+Ax+B}$.

Solution: As we showed in part **a.**, $f(x)$ has its only critical point at $x = -\dfrac{A}{2}$, so now we must have that $-\dfrac{A}{2} = 3$ so $A = -6$.
To find B, we use the fact that $(3, 1)$ is a point on the graph of $y = f(x)$ (so $f(3) = 1$).

$$\begin{aligned} 1 = f(3) &= e^{3^2+(-6)(3)+B} = e^{B-9} \\ \ln(1) &= \ln\left(e^{B-8}\right) \\ 0 &= B - 9 \\ B &= 9 \end{aligned}$$

Hence, $A = -6$ and $B = 9$.

Answer: $A =$ -6 and $B =$ 9

8. [7 points] Mr. R. DeVark discovers that there is a loud humming sound emanating from a tree in his backyard. The volume of the sound at any point in the yard is a function of the point's distance from the tree.

- Let $V(x)$ be the rate of change (in decibels per meter) of the volume of the sound where x is the distance (in meters) from the tree.
- Let $K(t)$ be the distance, in meters, of Mr. DeVark from the tree t seconds after he first notices the sound.

Assume that K is invertible and that V, K, and K^{-1} are differentiable.

a. [3 points] Give a practical interpretation of the equation $\displaystyle\int_{10}^{40} V(x)\,dx = -5$ in the context of this problem. *Remember to use a complete sentence and include units.*

Solution: The humming sound is 5 decibels louder at a point 10 meters away from the tree than it is at a point 40 meters from the tree.

b. [2 points] Which one of the following expressions represents the instantaneous rate of change (in decibels per second) of the volume at which Mr. DeVark hears the sound 30 seconds after he first notices the sound? Circle the <u>one</u> best answer.

$V(30)$	$V(30)K(30)$	$V'(30)K(30)$
$V'(30)$	$V(30)K'(30)$	$V'(30)K'(30)$
$V(K(30))$	$V(K(30))K(30)$	$V'(K(30))K(30)$
$V(K'(30))$	$\boxed{V(K(30))K'(30)}$	$V'(K(30))K'(30)$
$V'(K(30))$		
$V'(K'(30))$	$V(K'(30))K'(30)$	$V'(K'(30))K'(30)$

c. [2 points] Which of the following is the best interpretation of the equation $(K^{-1})'(15) = -2$? Circle the <u>one</u> best answer.

i. Between 15 and 15.5 seconds after Mr. DeVark notices the humming sound, he moves about 1 meter closer to the tree.

ii. It takes about 1 second for Mr. DeVark to go from being 15 meters away from the tree to 14.5 meters away from the tree.

iii. The volume of the humming sound is about 1 decibel lower at a point 15.5 meters from the tree than it is at a point 15 meters from the tree.

iv. When Mr. DeVark is 15 meters away from the tree, it is about 2 seconds before he notices the humming sound

v. The volume of the humming sound Mr. DeVark hears is about 1 decibel lower 15 seconds after he first notices it than 0.5 seconds later.

vi. When Mr. DeVark is 15 meters away from the tree, he moves about 2 meters closer to the tree in the next second.

9. [8 points] Suppose g is a twice differentiable function with continuous second derivative. Several values of the first and second derivatives of g are shown in the table below.

t	-8	-6	-4	-2	0	2	4	6	8
$g'(t)$	-11	-10	-8	0	9	16	17	14	7
$g''(t)$	1	0	2	5	4	2	0	-2	-4

Assume that between each pair of consecutive values of t shown in the table, each of $g'(t)$ and $g''(t)$ is either always strictly decreasing or always strictly increasing.

a. [2 points] Use the local linearization of $g'(t)$ near $t = 6$ to estimate $g'(5.8)$.

Solution: Let $L(t)$ be the local linearization of $g'(t)$ near $t = 6$.
Then $L(t) = g'(6) + g''(6)(t-6) = 14 - 2(t-6)$ and we find the estimate

$$g'(5.8) \approx L(5.8) = 14 - 2(5.8-6) = 14 + 0.4 = 14.4.$$

Answer: $g'(5.8) \approx$ 14.4

b. [1 point] Indicate whether the local linearization of $g'(t)$ near $t = 6$ gives an overestimate or an underestimate of the value of $g'(5.8)$. If there is not enough information to make this determination, circle "not enough information". You do not need to explain.
Answer: This estimate is an (circle one):

overestimate   underestimate   not enough information

c. [5 points] Let f be the quadratic function defined by $f(t) = t^2 - 4t + 6$. and let R be the function defined by $R(t) = f(t) - g(t)$. At what, if any, values of t does $R'(t)$ (the <u>derivative</u> of $R(t)$) attain its global extrema in the open interval $-8 < t < 8$?
For each answer blank, write NONE if $R'(t)$ does not attain a global extremum of that type on the open interval $-8 < t < 8$, and write NOT ENOUGH INFO if the t value(s) cannot be determined exactly. Use calculus to find and justify your answers, and be sure to show enough evidence to demonstrate that you have found where the global extrema occur.

Solution: Note: Since $R(t) = f(t) - g(t) = t^2 - 4t + 6 - g(t)$, we have $R'(t) = f'(t) - g'(t) = 2t - 4 - g'(t)$ and $R''(t) = f''(t) - g''(t) = 2 - g''(t)$.
To determine the global extrema of R', we first find the critical points of R', which are the points where R'' is undefined or equal to 0. Since both f'' and g'' are defined on the entire interval $-8 < t < 8$, there are no points in the interval where R'' is undefined. Since $R''(t) = 2 - g''(t)$, we see that $R''(t) = 0$ if and only if $g''(t) = 2$. Because g'' is continuous and strictly increasing or strictly decreasing between consecutive values of t in the table, we conclude that the only values of t in the interval $-8 < t < 8$ with $g''(t) = 2$ are $t = -4$ and $t = 2$.
Note that since R' is continuous, $\lim_{t\to -8^+} R'(t) = R'(-8) = -9$ and $\lim_{t\to 8^-} R'(t) = R'(8) = 25$.
Also note that $R''(t) > 0$ on the intervals $(-8, -4)$ and $(2, 8)$ (since $g''(t) < 2$ on these intervals), so $R'(t)$ is strictly increasing on both of these intervals.
We consider the value of R' at its critical points and the limit of R' at the ends of the open interval.

t	$f'(t)$	$g'(t)$	$R'(t)$
$\lim_{t\to -8^+}$	-20	-11	-9
$t = -4$	-12	-8	-4
$t = 2$	0	16	-16
$\lim_{t\to 8^-}$	14	-11	25

We conclude that (1) the global minimum value of $R'(t)$ on the interval $-8 < t < 8$ is -16, which occurs at $t = 2$, and (2) $R'(t)$ does not attain a global maximum on the interval $-8 < t < 8$. (The value of $R'(t)$ increases arbitrarily close to 25 as t approaches 8, but there is no value of t in the open interval $-8 < t < 8$ at which the value of $R'(t)$ is actually equal to 25.)

Answer: Global min(s) at $t =$ 2 and Global max(es) at $t =$ NONE

10. [10 points] For each of the questions below, circle all correct choices. If none of the choices are correct, circle NONE OF THESE.

You are not required to show your work on this page.

a. [2 points] Which of the following equations gives the tangent line to $y = \ln(3x+4)+1$ at $x = -1$? Circle all such equations.

i. $y = x + 2$

ii. $y = \dfrac{3}{3x+4} + 1$

iii. $\boxed{y = 3x + 4}$

iv. $y = 1$

v. $y = x + 4$

vi. NONE OF THESE

b. [2 points] Which of the following functions are antiderivatives of $f(x) = \cos(x)$? Circle all such functions.

i. $\frac{1}{2}(\cos(x))^2$

ii. $\boxed{\sin(x) + 5}$

iii. $\boxed{\cos\left(x - \frac{\pi}{2}\right)}$

iv. $\boxed{\ln\left(3e^{\sin(x)}\right)}$

v. $19 - \sin(x)$

vi. NONE OF THESE

c. [2 points] Which of the following limits equal 0? Circle all such expressions.

i. $\displaystyle\lim_{x\to\infty} \frac{e^x}{x}$

ii. $\boxed{\displaystyle\lim_{x\to\infty} \frac{e^{-x}}{x}}$

iii. $\displaystyle\lim_{x\to\infty} \sin(x)$

iv. $\displaystyle\lim_{x\to\infty} \frac{x^3 - 24x^2 + 188x - 480}{x^2 - 12x + 20}$

v. $\boxed{\displaystyle\lim_{x\to\infty} \frac{10000}{x^{1/1001}}}$

vi. NONE OF THESE

d. [2 points] For K a positive constant, which of the following limits equal K? Circle all such expressions.

i. $\displaystyle\lim_{h\to 0} \frac{K(1+h)^2 - K(1)^2}{h}$

ii. $\displaystyle\lim_{h\to 0} \frac{K\cos(h+2\pi) - K\cos(2\pi)}{h}$

iii. $\boxed{\displaystyle\lim_{h\to 0} \frac{K\sin(h+2\pi) - K\sin(2\pi)}{h}}$

iv. $\boxed{\displaystyle\lim_{h\to 0} \frac{e^{\ln(K)+h} - e^{\ln(K)}}{h}}$

v. $\boxed{\displaystyle\lim_{h\to 0} \frac{(1+h)^K - (1)^K}{h}}$

vi. NONE OF THESE

e. [2 points] For constants A and B, consider the function h defined by

$$h(t) = \begin{cases} (At)^2 - 48 & \text{if } t < 2 \\ Bt^3 & \text{if } t \geq 2. \end{cases}$$

Circle all pairs of values of A and B such that $h(t)$ is differentiable.

i. $A = 3, B = 3$

ii. $\boxed{A = 6, B = 12}$

iii. $\boxed{A = -6, B = 12}$

iv. $A = 0, B = 0$

v. $A = 18, B = 12$

vi. NONE OF THESE