On my honor, as a student,
I have neither given nor received
unauthorized aid on this academic work. **Initials:** _____

Do not write in this area

Math 115 — Second Midterm — April, 2020

Your Initials Only: _____ Your U-M ID # (not uniqname): _____

Instructor Name: _____ Section #: _____

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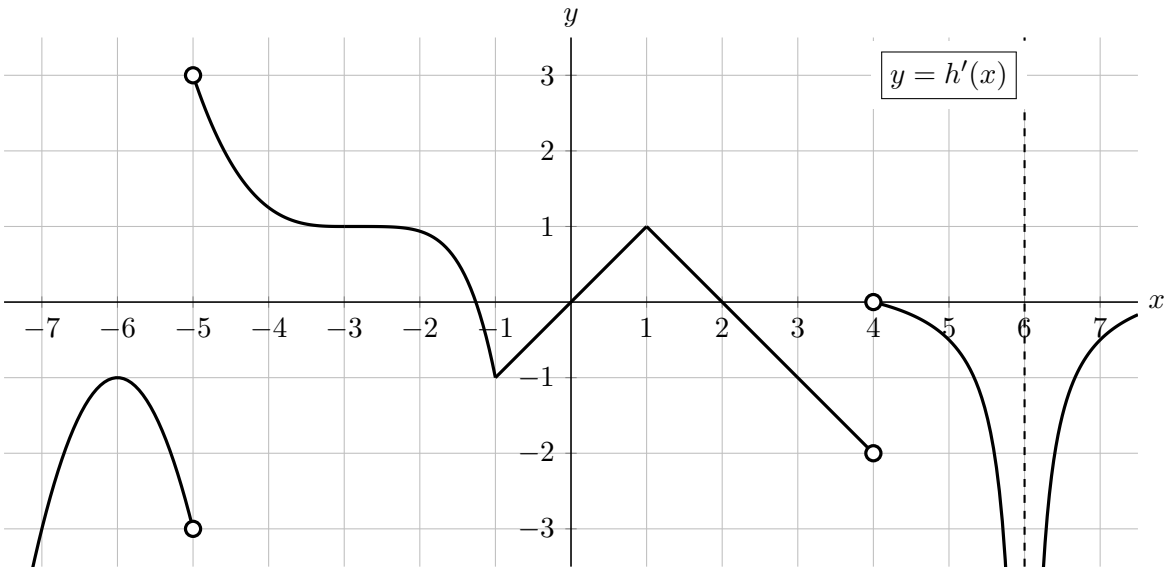
1. **Do not open this exam until you are told to do so.**
2. **Do not write your name anywhere on this exam.**
3. This exam has 10 pages including this cover. There are 9 problems.
   Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
4. Do not separate the pages of this exam. If they do become separated, write your UMID (not name) on every page and point this out to your instructor when you hand in the exam.
5. Note that the back of every page of the exam is blank, and, if needed, you may use this space for scratchwork. Clearly identify any of this work that you would like to have graded.
6. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
7. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it.
8. The use of any networked device while working on this exam is <u>not</u> permitted.
9. You may use any one calculator that does not have an internet or data connection except a TI-92 (or other calculator with a "qwerty" keypad). However, you must show work for any calculation which we have learned how to do in this course.
   You are also allowed two sides of a single $3'' \times 5''$ notecard.
10. For any graph or table that you use to find an answer, be sure to sketch the graph or write out the entries of the table. In either case, include an explanation of how you used the graph or table to find the answer.
11. Include units in your answer where that is appropriate.
12. Problems may ask for answers in *exact form*. Recall that $x = \sqrt{2}$ is a solution in exact form to the equation $x^2 = 2$, but $x = 1.41421356237$ is <u>not</u>.
13. **Turn off all cell phones, smartphones, and other electronic devices**, and remove all headphones, earbuds, and smartwatches. Put all of these items away.
14. You must use the methods learned in this course to solve all problems.

Problem	Points	Score
1	12	
2	11	
3	12	
4	12	
5	10	

Problem	Points	Score
6	10	
7	9	
8	5	
9	9	
Total	90	

1. [12 points] A function $h(x)$ is defined and continuous on $(-\infty, \infty)$. A portion of the graph of $h'(x)$, **the derivative of** $h(x)$, is shown below. Note that $h'(x)$ has a vertical asymptote at $x = 6$.



In each part **a.**–**f.** below, select **all** correct choices.

a. [2 points] At which of the following value(s) does $h(x)$ have a critical point?

$x = -6$ $x = -3$ $x = 0$ $x = 1$ NONE OF THESE

b. [2 points] At which of the following value(s) does $h(x)$ have a local minimum?

$x = -5$ $x = -1$ $x = 2$ $x = 6$ NONE OF THESE

c. [2 points] At which of the following value(s) does $h(x)$ have an inflection point?

$x = -6$ $x = -5$ $x = -3$ $x = 6$ NONE OF THESE

d. [2 points] On which of the following interval(s) is $h(x)$ increasing on the entire interval?

$(-5, -3)$ $(-1, 1)$ $(6, 7)$ NONE OF THESE

e. [2 points] On which of the following interval(s) is $h(x)$ concave down on the entire interval?

$(-7, -5)$ $(-5, -3)$ $(-1, 1)$ NONE OF THESE

f. [2 points] On which of the following interval(s) is $h''(x)$ decreasing on the entire interval?

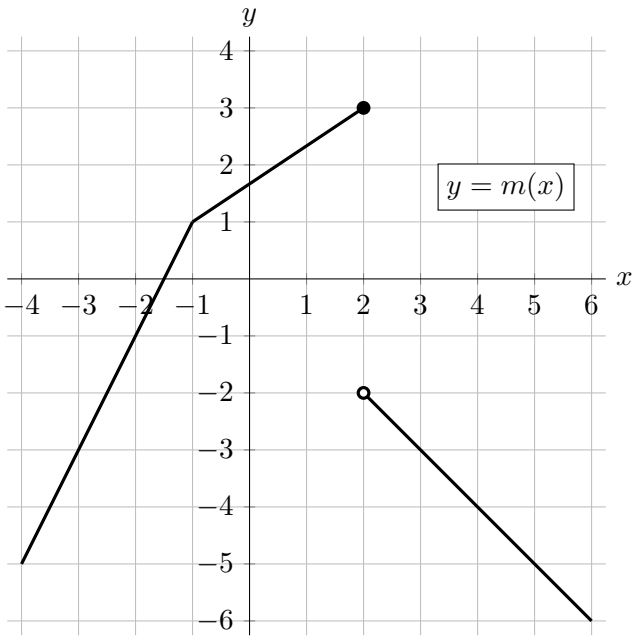
$(-7, -5)$ $(-5, -3)$ $(-1, 1)$ NONE OF THESE

2. [11 points]

The function $m(x)$ is defined on $(-\infty, \infty)$.

A portion of the graph of the function $m(x)$ is shown.

Note that $m(x)$ is linear on the intervals $(-4, -1)$, $(-1, 2)$ and $(2, 6)$.



a. [8 points] Evaluate each of the following quantities **exactly**, or write DNE if the value does not exist. You do not need to show work, but limited partial credit may be awarded for work shown. Your answers should not contain the letter m, but do not need to be fully simplified.

i. [2 points] Let $v(x) = x^3 m(x)$. Compute $v'(4)$.

Answer: $v'(4) =$ _____

ii. [2 points] Let $u(x) = 5m(x-1) + 8$. Compute $u'(3)$.

Answer: $u'(3) =$ _____

iii. [2 points] Let $w(x) = \dfrac{1}{m(x)}$. Compute $w'(-3)$.

Answer: $w'(-3) =$ _____

iv. [2 points] Let $r(x) = \sin(\pi m(x))$. Compute $r'(-2)$.

Answer: $r'(-2) =$ _____

b. [3 points] Suppose $j(x)$ is a function whose **derivative** is given by the above graph (i.e. $j'(x) = m(x)$). Find a formula for $Q(x)$, the quadratic approximation of $j(x)$ at $x = 5$, assuming $j(5) = 4$.

Answer: $Q(x) =$ _____

3. [12 points]
Suppose $h(x)$ is a continuous function defined for all real numbers x. The **derivative** and **second derivative** of $h(x)$ are given by

$$h'(x) = (x-13)^2(x+4)^{3/7} \quad \text{and} \quad h''(x) = \frac{17(x-13)(x+1)}{7(x+4)^{4/7}}.$$

a. [6 points] Find the x-coordinates of all local extrema of $h(x)$. If there are none of a particular type, write NONE. Use calculus to find and justify your answers, and be sure to show enough evidence to demonstrate that you have found all local extrema.

Answer: Local max(es) at x = _____ Local min(s) at x = _____

b. [6 points] Find the x-coordinates of all inflection points of $h(x)$. If there are none, write NONE. Use calculus to find and justify your answers, and be sure to show enough evidence to demonstrate that you have found all inflection points.

Answer: Inflection Point(s) at x = _____

4. [12 points] Isabelle is a bee keeper who wants to sell honey at the local farmers market. Let $y = H(d)$ be the amount of honey, in pounds, that Isabelle will sell in a month if she charges d dollars per pound of honey. The functions $H(d)$ and $H'(d)$ are defined and differentiable for all $d \geq 0$. Some values are given in the table below.

d	5.00	5.75	6.50	7.25	8.00	8.75
$H(d)$	59	52	46	38	29	23
$H'(d)$	-10.4	-9.1	-7.8	-11.0	-12.2	-7.6

Assume that $H(d)$ is decreasing and that between each pair of consecutive values of d given in the table, $H'(d)$ is either always increasing or always decreasing.

a. [3 points] Write a formula for the linear approximation $L(d)$ of $H(d)$ near $d = 6.50$, and use it to estimate the amount of honey, in pounds, Isabelle will sell if she charges \$6.30 per pound.

Answer: $L(d) =$ _____

Answer: $\approx$ _____

b. [2 points] Is your estimate from the previous part an overestimate, an underestimate, neither, or is there not enough information to decide? Briefly explain your answer.

c. [3 points] Write a formula for the linear approximation $K(y)$ of $(H^{-1})(y)$ near $y = 31$.

Answer: $K(y) =$ _____

d. [2 points] Use the table to approximate $H''(8.75)$.

Answer: $H''(8.75) \approx$ _____

e. [2 points] The hypotheses of the Mean Value Theorem are satisfied for $H(d)$ on the interval $[5.00, 5.75]$. The conclusion of the theorem then tells you that there is a c in the interval $[5, 5.75]$ so that

_____ $=$ _____

5. [10 points] Let $P = F(t)$ be the size, in thousands of people, of a certain band's fan club t years after the beginning of 2020. Formulas modeling $F(t)$ and $F'(t)$, the **derivative** of $F(t)$, are given below.

$$F(t) = 175 + 35(t^3 - 7t^2 + 13t - 5)e^{-t} \quad \text{and} \quad F'(t) = -35(t-1)(t-3)(t-6)e^{-t}.$$

In both parts below, you must use calculus to find your answers, and be sure to show enough evidence to fully justify your answers. For each answer blank, write NONE if appropriate.

a. [5 points] During the first two years after the beginning of 2020 (i.e. for $0 \leq t \leq 2$), when will the band's fan club have the largest and the smallest number of members?

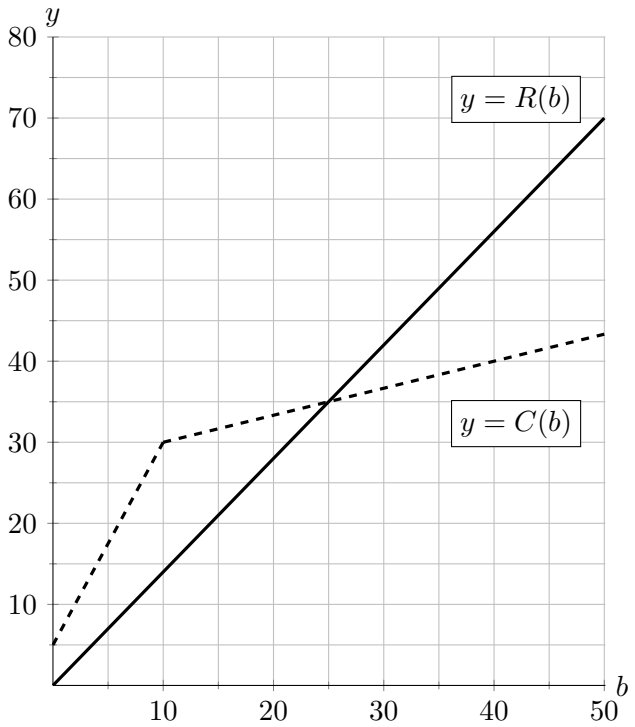
Answer: Largest at t = _____ Smallest at t = _____

b. [5 points] After the beginning of 2022 (i.e. for $t \geq 2$), what are the largest and smallest number of members the band's fan club will have?

Answer: Largest number of members, in thousands: _____

Answer: Smallest number of members, in thousands: _____

6. [10 points] A local bakery makes and sells bagels. When they make and sell b bagels in a given day, their cost is $C(b)$ dollars and their revenue is $R(b)$ dollars. Below, $R(b)$ is graphed as solid line, while $C(b)$ is graphed as a dashed line. Note that the bakery only has the capability to make up to 50 bagels each day.



a. [1 point] What are the company's fixed costs, in dollars?

Answer: _____

b. [2 points] What is the selling price, in dollars, of each bagel?

Answer: _____

c. [2 points] Find the marginal cost, in dollars per bagel, at $b = 20$ bagels.

Answer: _____

d. [2 points] Estimate the bakery's daily profit, in dollars, if they produce and sell 40 bagels.

Answer: _____

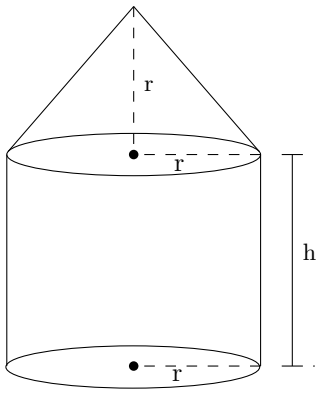
e. [2 points] How many bagels should the bakery produce and sell each day if they want to maximize their profit?

Answer: _____

f. [1 point] How many bagels would the bakery have to produce and sell each day to **minimize** their profit (that is, maximize their losses)?

Answer: _____

7. [9 points]



A city is in the planning stages of building a shed to store road salt. One design being considered is shown above. The sides would be a cylinder of radius r feet and height h feet, and the roof would be a cone in which both the radius and height are equal to r feet. (The city does not need to build a floor.) The cost of the materials for this shed, in dollars, is

$$4\pi r^2 + 4\pi rh.$$

If the city wants to spend \$20,000 on materials, what values of r and h will maximize the volume of the shed? Give your answers to at least two decimal places, and be sure to find and justify your answers using calculus.

Note that the volume of a cone with radius R and height H is $\dfrac{1}{3}\pi R^2 H$.

8. [5 points]
A curve is implicitly defined by the equation

$$\ln(kx) - 3xy^2 = \pi,$$

where k is a constant. Compute $\dfrac{dy}{dx}$. Your answer may include k. Show every step of your work.

Answer: $\dfrac{dy}{dx} =$ _____

9. [9 points] The function $g(x)$ is given by the equation

$$g(x) = \begin{cases} 3|x+2| - 8x - 11 & x \leq -1 \\ x^2 - 3x - 4 & -1 < x < 2 \\ 12(x-10)^{1/3} + 2x + 14 & x \geq 2. \end{cases}$$

You must show work for parts a–d of this problem.

a. [1 point] Is $g(x)$ continuous at -1?

b. [2 points] Is $g(x)$ differentiable at -1?

c. [1 point] Is $g(x)$ continuous at 2?

d. [2 points] Is $g(x)$ differentiable at 2?

e. [3 points] List **all** points at which $g(x)$ is not differentiable.