Math 115 — First Midterm — February 23, 2021

1. This exam has 8 pages including this cover. There are 10 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer mathematical questions about exam problems during the exam.
3. You must use the methods learned in this course to solve all problems.
4. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it.
5. Problems may ask for answers in *exact form*. Recall that $x = \frac{1}{3}$ is an exact answer to the equation $3x = 1$, but $x = 0.33333$ is not.
6. You must write your work and answers on **blank, white, physical paper**.
7. You must write your **initials and UMID**, but not your name or uniqname, in the upper right corner of every page of work. Make sure that it is visible in all scans or images you submit.
8. Make sure that all pages of work have the relevant problem number clearly identified.

Problem	Points	Score
1	3	
2	12	
3	11	
4	12	
5	10	
6	9	

Problem	Points	Score
7	9	
8	8	
9	7	
10	14	
Total	95	

1. [3 points] **There is work to submit for this problem. Read it carefully.**

- You may use your one pre-written page of notes, on an 8.5" by 11" standard sheet of paper, with whatever you want handwritten (not typed) on both sides.
- You are not allowed to use any other resources, including calculators, other notes, or the book.
- You may not use any electronic device or the internet, except to access the Zoom meeting for the exam, to access the exam file itself, to submit your work, or to report technological problems via the Google forms we will provide to do so. The one exception is that you may use headphones (e.g. for white noise) if you prefer, though please note that you need to be able to hear when the end of the exam is called in the Zoom meeting.
- You may not use help from any other individuals (other students, tutors, online help forums, etc.), and may not communicate with any other person about the exam until **11pm on Tuesday** (Ann Arbor time).
- The one exception to the above policy is that you may contact the proctors in your exam room via the chat in Zoom if needed.
- Violation of any of the policies above may result in a score of zero for the exam, and, depending on the violation, may result in a failing grade in the course.

As your submission for this problem, you must write "I agree," and write your initials and UMID number to signify that you understand and agree to this policy. By doing this and submitting it to Gradescope, you are attesting that you have not violated this policy.

2. [12 points]

a. [6 points] Consider the given table of values for the function $j(u)$.

u	1	2	3
$j(u)$	6		54

i. Supposing that $j(u)$ is a linear function, fill in the missing entry in the table. Show your work.

ii. Supposing that $j(u)$ is an exponential function, fill in the missing entry in the table. Show your work.

b. [6 points] A radioactive substance decays exponentially in such a way that, if you have some amount of it, then after 15.2 days you will only have a third as much of it remaining. If I have 110 grams of this substance today, how long will I have to wait until I only have 3 grams remaining? *Show every step of your work, and give your final answer in exact form.*

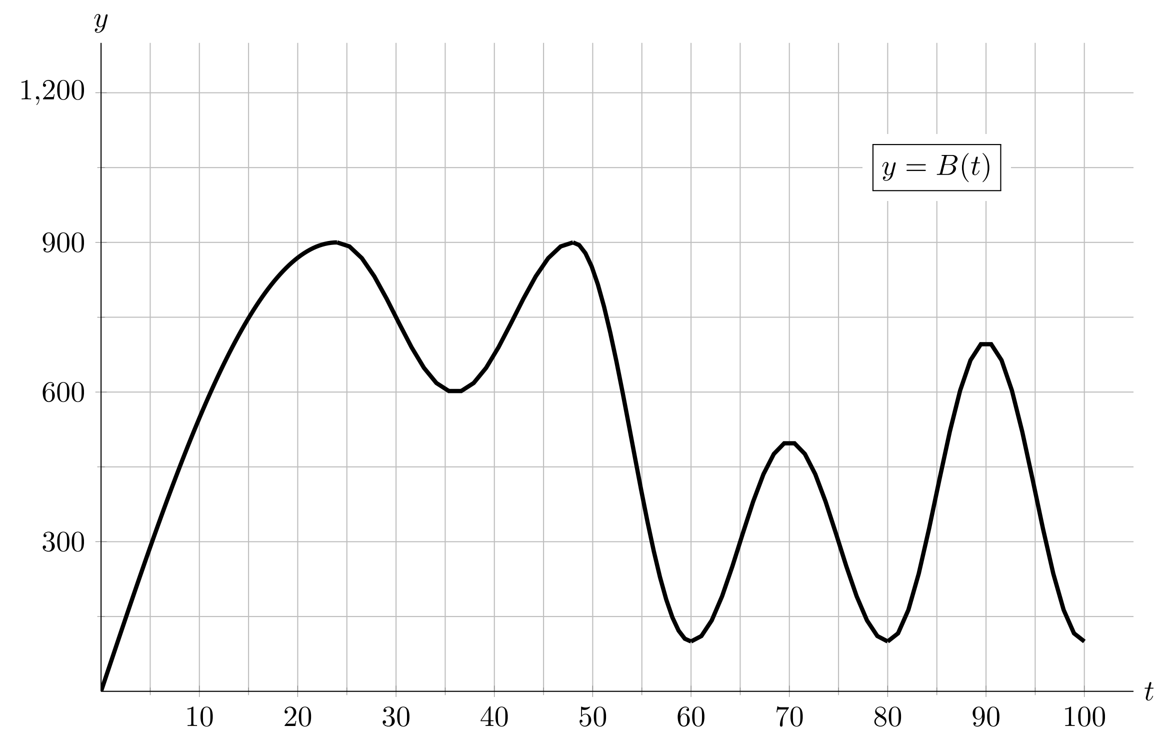
3. [11 points] A pilot is flying in an air show. Let $A(t)$ be her altitude, in feet (ft) above the ground, t seconds (sec) after takeoff. Some values of $A(t)$ are shown in the table below, and there is one missing value, denoted by "?".

t	5	22	23	60	60.1	70
$A(t)$	300	1100	1400	400	?	1200

a. [3 points] Use the table to give the best possible estimate of $A'(22)$. Make sure to include the relevant units as part of your answer.

b. [3 points] Suppose that $A'(60) = 550$. Give an approximate value for the missing entry in the table. Make sure to include the relevant units as part of your answer.

c. [5 points] The pilot flies in a different air show a week later. Let $B(t)$ be her altitude, in feet (ft) above the ground, t seconds (sec) after takeoff. A graph of $B(t)$ is shown below.



Let the quantities I-V be defined as follows:

I. The number 0.

II. The pilot's average velocity, in ft/sec, between $t = 15$ and $t = 50$.

III. The pilot's instantaneous velocity, in ft/sec, at $t = 55$.

IV. The pilot's average velocity, in ft/sec, between $t = 50$ and $t = 90$.

V. The pilot's instantaneous velocity, in ft/sec, at $t = 85$.

List the quantities I-V in increasing order.

4. [12 points] Parts **a.** and **b.** below are unrelated.

a. [6 points] Suppose that the temperature in Staunton, Virginia, in degrees Fahrenheit (°F), can be modeled by a sinusoidal function $S(t)$ where t is the time in months since January 1. Note that, for example, August 1 is seven months after January 1. A formula for $S(t)$ is

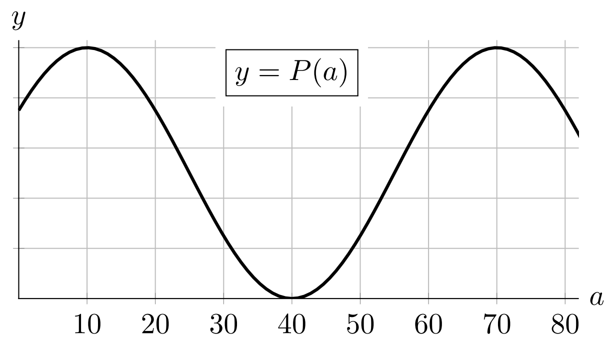
$$55 - 21\cos\left(\frac{\pi}{6}t\right),$$

i. Using this model, what is the coldest temperature in Staunton?

ii. Using this model, what is the average temperature over the entire year?

iii. At what time t does the temperature first reach "room temperature" (68°F)? *Give your final answer in exact form.*

b. [6 points] Suppose that a probe lands on some planet other than Earth, and that its recorded temperature, in degrees Fahrenheit, can be modeled by a sinusdoidal function $P(a)$ where a is the time in years since the probe landed. Note that the scale on the y-axis is unknown.



When the temperature is too cold, the probe is in a state of hibernation. The first time it enters hibernation is at $a = 27$.

i. At what time a does the probe leave hibernation?

ii. What is the period of $P(a)$?

iii. Use the period you found to calculate the next time at which the probe will enter hibernation.

5. [10 points] Let us consider the following functions, which concern the productivity of a soybean farm. Bushels are a unit of volume often used to measure a farm's yield.

- Let $Y(b)$ be the yield, in bushels of soybeans, of the farm in the year 2019 when it is infested with b beetles.
- Let $R(s)$ be the revenue, in dollars, of the farm in the year 2019 when it yields s bushels of soybeans.

The functions $Y(b)$ and $R(s)$ are differentiable and invertible.

a. [2 points] Use a complete sentence to give a practical interpretation of the equation

$$R(Y(1{,}200)) = 75{,}000.$$

b. [4 points] Write a single equation representing the following statement in terms of the functions Y, R, and/or their inverses:

If there are 1,600 *beetles, then the farm yields* 200 *bushels of soybeans fewer than are necessary for a revenue of* \$64,000 in the year 2019.

c. [4 points] Complete the following sentence to give a practical interpretation of the equation

$$Y'(1{,}000) = -0.1.$$

If the beetle population was 1,000 *rather than* 950...

6. [9 points] A metal bar is unevenly heated, and a laser thermometer is used to measure its temperature at various points. Let $T(q)$ be the temperature of the bar, in degrees Celsius, q feet from its leftmost end. Some values of $T(q)$ are shown in the table below.

q	1	2	3	4	5	6	7	8	9
$T(q)$	40	70	90	80	60	90	130	100	60

a. [3 points] For which of the following intervals of q-values might the function $T'(q)$ be positive for the entire interval? Give your answer as a list of one or more intervals, or write NONE.

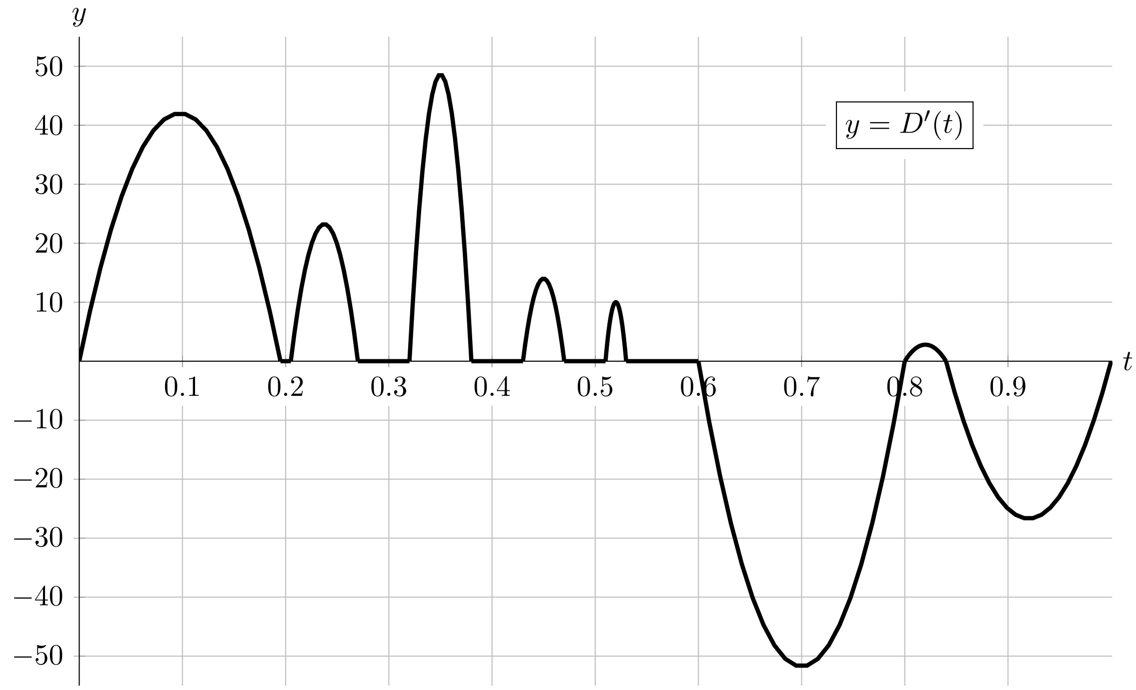
$(1, 3)$ $(4, 6)$ $(5, 7)$ $(7, 9)$

b. [3 points] For which of the following intervals of x-values might the function $T(q)$ be concave up for the entire interval? Give your answer as a list of one or more intervals, or write NONE.

$(1, 3)$ $(4, 6)$ $(5, 7)$ $(7, 9)$

c. [3 points] What is the average rate of change of $T(q)$ on the interval $2 \leq q \leq 7$? Include units in your answer.

7. [9 points] A pizza delivery driver works for a pizzeria on Main Street, which is a long, straight road. The driver tracks her location with her phone while driving a route on Main Street. Let $D(t)$ be her distance from her pizzeria, in miles, at time t hours after noon. Below is a portion of the graph of $D'(t)$, the **derivative** of $D(t)$.



a. [2 points] On which of the following intervals of t is the driver getting closer to her pizzeria for the entire interval? Give your answer as a list of one or more intervals, or write NONE.

$(0.1, 0.2)$ $(0.2, 0.3)$ $(0.6, 0.8)$ $(0.8, 1)$

b. [3 points] The speed limit in the driver's hometown is 40 miles per hour. How many different times does she *begin* to drive over the speed limit?

c. [2 points] At which of the following times is the driver farthest from her pizzeria? Write the one best answer.

$t = 0.1$ $t = 0.35$ $t = 0.5$ $t = 0.6$ $t = 0.7$

d. [2 points] Write the number of the the sentence below that best describes the driver's behavior on the interval $0.2 \leq t \leq 0.5$.

1. *The driver keeps returning to the pizzeria to pick up more pizza.*
2. *The driver is driving on a highway without any traffic.*
3. *The driver stops at a series of red lights.*
4. *The driver is driving in circles, looking for a place to park.*

8. [8 points]

a. [3 points] Let

$$L(q) = \frac{\sin(q)}{1+3q}.$$

Suppose k is a nonzero constant. Write an explicit expression for the average rate of change of L between $q = 5$ and $q = 5 + k$.

Your answer should not involve the letter L. Do not attempt to simplify your expression.

Draw a box around your final answer.

b. [5 points] Let

$$P(b) = (\ln(b))^{\tan(b)}.$$

Use the limit definition of the derivative to write an explicit expression for $P'(3)$.

Your answer should not involve the letter P. Do not attempt to evaluate or simplify the limit.

Draw a box around your final answer.

9. [7 points]

We consider formulas for four different rational functions. Let the formulas I-IV be defined as follows:

I. $\dfrac{x^5(x+1)(x-2)}{x+2}$

II. $\dfrac{x^4(x+1)(x-2)(x-8)}{(x-9)(x-3)}$

III. $\dfrac{(x-9)(x+1)}{x-8}$

IV. $\dfrac{(x-3)(x+3)}{x-9}$

We describe three functions. Match each function below with the formula I-IV that could possibly be the formula for that function. Each function below matches with *exactly one* of the formulas above.

A. The function $g(x)$ is such that $\dfrac{g(x)}{x^5}$ diverges to ∞ as $x \to \infty$.

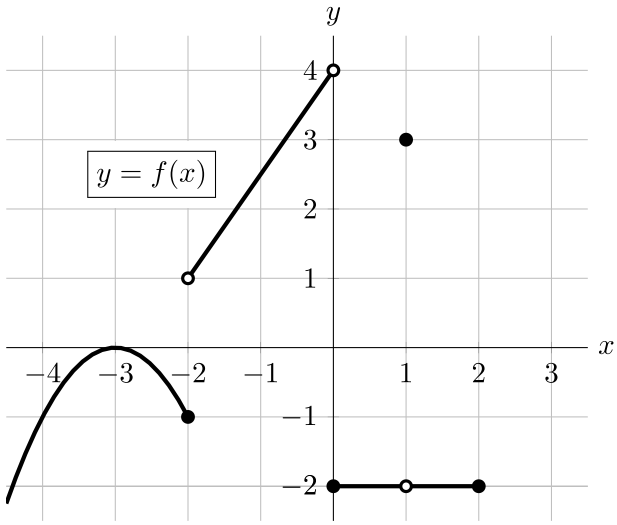
B. The function $h(x)$ is such that the function

$$S(t) = \begin{cases} \sin(2\pi x) & x < 3 \\ h(x) & x \geq 3 \end{cases}$$

is continuous at $x = 3$.

C. The function $f(x)$ is such that $f(x+3)$ has a vertical asymptote at $x = 5$.

10. [14 points] The graph of the function $f(x)$ is shown below.



For **a.–b.**, give your answers as a list of one or more of the given numbers, or write NONE

a. [2 points] For which of the values $c = -3, -2, -1, 0, 1$ is $f(x)$ continuous at $x = c$?

b. [2 points] For which of the values $c = -3, -2, -1, 0, 1$ is $\lim_{x \to c^-} f(x) = f(c)$?

For **c.–g.**, use the graph of the function $f(x)$ to evaluate each of the expressions below. If a limit diverges to ∞ or $-\infty$ or if the limit does not exist for any other reason, write "DNE." If there is not enough information to evaluate the expression, write "Not enough information."

c. [2 points] $\lim_{x \to 0} f(x)$

d. [2 points] $\lim_{x \to 1} f(x)$

e. [2 points] $\lim_{h \to 0} \dfrac{f(-1+h) - f(-1)}{h}$

f. [2 points] $\lim_{x \to 3^+} 4f(x-5) - 1$

g. [2 points] $\lim_{x \to -3} f(f(x))$