Math 115 — Second Midterm — March 22, 2022

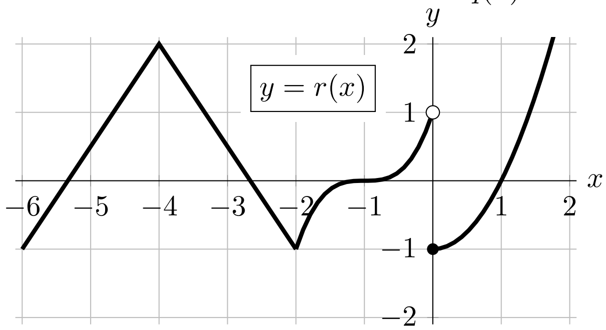
EXAM SOLUTIONS

1. **Do not open this exam until you are told to do so.**
2. **Do not write your name anywhere on this exam.**
3. This exam has 11 pages including this cover. There are 10 problems.
   Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
4. Do not separate the pages of this exam. If they do become separated, write your UMID (not name) on every page and point this out to your instructor when you hand in the exam.
5. The back of every page of the exam is blank, and, if needed, you may use this space for scratchwork. Clearly identify any of this work that you would like to have graded.
6. Read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
7. Show an appropriate amount of work for each problem, so that graders can see not only your answer but how you obtained it.
8. You must use the methods learned in this course to solve all problems.
9. You are not allowed to use a calculator of any kind on this exam.
   You are allowed notes written on two sides of a $3'' \times 5''$ note card.
10. Problems may ask for answers in *exact form*. Recall that $x = \sqrt{2}$ is a solution in exact form to the equation $x^2 = 2$, but $x = 1.41421356237$ is not.
11. **Turn off all cell phones, smartphones, and other electronic devices**, and remove all headphones, earbuds, and smartwatches. Put all of these items away. The use of any networked device while working on this exam is not permitted.

Problem	Points	Score
1	9	
2	7	
3	10	
4	8	
5	15	

Problem	Points	Score
6	9	
7	5	
8	8	
9	11	
10	8	
Total	90	

1. [9 points] A portion of a graph of the function $r(x)$, whose domain is $(-\infty, \infty)$ is shown below to the left. The function $r(x)$ is linear on the intervals $[-6, -4]$ and $[-4, -2]$. A table of values for a differentiable and invertible function $q(x)$ and its derivative $q'(x)$ are shown below to the right.



x	-3	-2	-1	0	1	2	3
$q(x)$	14	10	3	2	-5	-6	-15
$q'(x)$	-10	-12	-4	0	-2	-5	-6

Find the **exact** values of the quantities in parts **a.-d.**, whenever possible. Write NEI if there is not enough information to do so, or write DNE if the value does not exist. Your answers should not include the letters q or r but you do not need to simplify your numerical answers. Show your work.

a. [1 point] Find $r'(-4)$.

Solution: The graph of $r(x)$ has a sharp corner at $x = -4$ so $r'(-4)$ does not exist. (The slope from the right is 1.5 while the slope from the left is -1.5.)

Answer: $r'(-4) =$ **DNE**

b. [2 points] Find $(q^{-1})'(-6)$.

Solution: We have

$$(q^{-1})'(x) = \frac{1}{q'(q^{-1}(x))}$$

so

$$(q^{-1})'(-6) = \frac{1}{q'(q^{-1}(-6))} = \frac{1}{q'(2)} = \frac{1}{-5} = -\frac{1}{5}.$$

Answer: $(q^{-1})'(-6) =$ $-1/5$

c. [3 points] Let $J(x) = e^{q(x)}$. Find $J'(1)$.

Solution: Applying the chain rule, we find

$$J'(x) = e^{q(x)} q'(x)$$

so

$$J'(1) = e^{q(1)} q'(1) = e^{-5}(-2) = -2e^{-5}.$$

Answer: $J'(1) =$ $-2e^{-5}$

d. [3 points] Let $D(x) = r(x)q(2x + 4)$. Find $D'(-3)$.

Solution: Applying the product and chain rules, we find

$$D'(x) = r'(x)q(2x + 4) + r(x)q'(2x + 4)(2)$$

so

$$D'(-3) = r'(-3)q(-2) + r(-3)q'(-2)(2) = -\frac{3}{2} \cdot 10 + \frac{1}{2} \cdot (-12) \cdot 2 = -15 - 12 = -27.$$

Answer: $D'(-3) =$ -27

2. [7 points] A table of values for a differentiable and invertible function $q(x)$ and its derivative $q'(x)$ are shown below. Note that this is the same function q as on the previous page. However, you do not need your work or answers from the previous page to do this problem.

x	-3	-2	-1	0	1	2	3
$q(x)$	14	10	3	2	-5	-6	-15
$q'(x)$	-10	-12	-4	0	-2	-5	-6

Let $\mathcal{C}$ be the curve defined implicitly by the equation

$$xy^2 + \sin(2\pi q(x)) = 6e^{y-4} + 10.$$

a. [1 point] Exactly one of the following points (x, y) lies on the curve $\mathcal{C}$. Circle that one point.

$(-2, 1)$ $\qquad$ $\boxed{(1, 4)}$ $\qquad$ $(0, 4)$ $\qquad$ $(0, 10)$

b. [6 points] Find an equation for the tangent line to the curve $\mathcal{C}$ at the point you chose in part **a**. Make sure to show your work clearly.

Solution: The slope of this tangent line is equal to $\left.\frac{dy}{dx}\right|_{(x,y)=(1,4)}$. To compute this, we first take the derivative with respect to x of both sides of the given equation for $\mathcal{C}$ and solve for $\frac{dy}{dx}$.

$$\begin{aligned}
\frac{d}{dx}\left(xy^2 + \sin\left(2\pi q(x)\right)\right) &= \frac{d}{dx}\left(6e^{y-4} + 10\right) \\
y^2 + 2xy\frac{dy}{dx} + 2\pi\cos(2\pi q(x))q'(x) &= 6e^{y-4}\frac{dy}{dx} \\
(2xy - 6e^{y-4})\frac{dy}{dx} &= -y^2 - 2\pi\cos(2\pi q(x))q'(x) \\
\frac{dy}{dx} &= \frac{-y^2 - 2\pi\cos(2\pi q(x))q'(x)}{2xy - 6e^{y-4}}.
\end{aligned}$$

So at the point $(1, 4)$,

$$\begin{aligned}
\left.\frac{dy}{dx}\right|_{(x,y)=(1,4)} &= \frac{-y^2 - 2\pi\cos(2\pi q(x))q'(x)}{2xy - 6e^{y-4}} \\
&= \frac{-(4)^2 - 2\pi\cos(2\pi q(1))q'(1)}{2(1)(4) - 6e^{4-4}} \\
&= \frac{-16 - 2\pi\cos(-10\pi)(-2)}{8 - 6} \\
&= \frac{-16 + 4\pi}{2} \\
&= 2\pi - 8.
\end{aligned}$$

Therefore, an equation for the tangent line to the curve $\mathcal{C}$ at the point $(1, 4)$ is

$$y = 4 + (2\pi - 8)(x - 1)$$

Answer: $y =$ $4 + (2\pi - 8)(x - 1)$

3. [10 points] Suppose $B(x)$ is a continuous function defined for all real numbers x. The derivative and second derivative of $B(x)$ are given by

$$B'(x) = e^{-x}x(\sqrt[3]{x-2})^2 \quad \text{and} \quad B''(x) = \frac{-e^{-x}(x-3)(3x-2)}{3\sqrt[3]{x-2}}.$$

a. [5 points] Find the exact x-coordinates of all local minima and local maxima of $B(x)$. If there are none of a particular type, write NONE. You must use calculus to find and justify your answers. Be sure your conclusions are clearly stated and that you show enough evidence to support them.

Solution: We begin by finding the critical points of $B(x)$.
Note that $B'(x) = 0$ when $x = 0$ and $x = 2$. Both of these x-coordinates are in the domain of $B(x)$, so they are both critical points of B.
Since $B'(x)$ is defined for all x, there are no points where $B(x)$ is not differentiable, so the only critical points of $B(x)$ are $x = 0$ and $x = 2$. We next apply the First Derivative Test to classify these critical points.

(checking signs of $B'(x)$)	$x < 0$	$0 < x < 2$	$2 < x$
e^{-x}	$+$	$+$	$+$
x	$-$	$+$	$+$
$(\sqrt[3]{x-2})^2$	$+$	$+$	$+$
$B'(x) = e^{-x}x(\sqrt[3]{x-2})^2$	$+\cdot-\cdot+=-$	$+\cdot+\cdot+=+$	$+\cdot+\cdot+=+$

This gives the following number line for $B'(x)$:

$B'(x)$	$+\cdot-\cdot+=-$		$+\cdot+\cdot+=+$		$+\cdot+\cdot+=+$
x		0		2	

By the First Derivative Test, since $B(x)$ is continuous, $B(x)$ has a local minimum at $x = 0$ and no local maxima.

Answer: Local min(s) at $x =$ 0 and Local max(es) at $x =$ **None**

b. [5 points] Find the exact x-coordinates of all inflection points of $B(x)$, or write NONE if there are none. You must use calculus to find and justify your answers. Be sure your conclusions are clearly stated and that you show enough evidence to support them.

Solution: To find candidate inflection points, we check when $B''(x) = 0$ and $B''(x)$ does not exist. $B''(x) = 0$ when $x = 3$ and $x = \frac{2}{3}$. $B''(x)$ does not exist when $x = 2$. These are all in the domain of $B(x)$ so $x = \frac{2}{3}, 2, 3$ are the candidate inflection points of $B(x)$. We now consider the sign of $B''(x)$ to determine which of these candidates are actually inflection points.

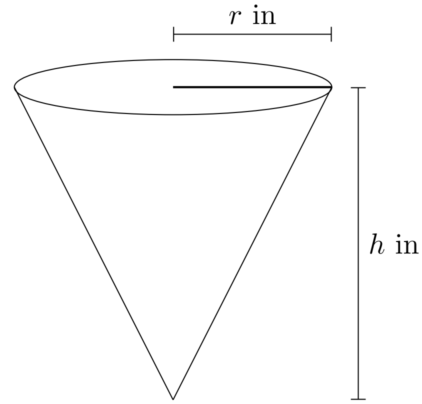
(checking signs of $B''(x)$)	$x < \frac{2}{3}$	$\frac{2}{3} < x < 2$	$2 < x < 3$	$3 < x$
$-e^{-x}$	$-$	$-$	$-$	$-$
$(x-3)$	$-$	$-$	$-$	$+$
$(3x-2)$	$-$	$+$	$+$	$+$
$\sqrt[3]{x-2}$	$-$	$-$	$+$	$+$
$B''(x) = \frac{-e^{-x}(x-3)(3x-2)}{3\sqrt[3]{x-2}}$	$\frac{-\cdot-\cdot-}{-} = +$	$\frac{-\cdot-\cdot+}{-} = -$	$\frac{-\cdot-\cdot+}{+} = -$	$\frac{-\cdot+\cdot+}{+} = -$

This gives the following number line for $B''(x)$:

$B''(x)$	$\frac{-\cdot-\cdot-}{-} = +$		$\frac{-\cdot-\cdot+}{-} = -$		$\frac{-\cdot-\cdot+}{-} = -$		$\frac{-\cdot+\cdot+}{+} = -$
x		2/3		2		3	

Because the sign of the second derivative changes at each of the candidate inflection points, all of $x = \frac{2}{3}, 2, 3$ are indeed inflection points.

Answer: Inflection point(s) at $x =$ $\frac{2}{3}, 2, 3$



4. [8 points]
Sunny and Tyrell own an ice cream shop together. They want to sell waffle cones in the usual shape of a cone, as shown on the right. The cost, in dollars, of a waffle cone with radius r inches and height h inches is

$$\frac{r}{2}\left(\sqrt{h^2+r^2}\right).$$

Sunny and Tyrell want to spend exactly \$5 on a waffle cone that can fit the most ice cream (i.e has the largest volume).

Note that the volume of a cone of radius r and height h is $\dfrac{\pi r^2 h}{3}$.

a. [3 points] Write a formula for h in terms of r if the cone costs \$5.

Solution: Because Sunny and Tyrell want to spend exactly \$5 on a waffle cone, we must have $\frac{r}{2}\left(\sqrt{h^2+r^2}\right) = 5$. Solving this equation for h, we find

$$\begin{aligned}\sqrt{h^2+r^2} &= \frac{10}{r}\\ h^2+r^2 &= \frac{100}{r^2}\\ h^2 &= \frac{100}{r^2}-r^2\\ h &= \sqrt{\frac{100}{r^2}-r^2}.\end{aligned}$$

Answer: $h =$ $\sqrt{\dfrac{100}{r^2}-r^2}$

b. [2 points] Write a formula for the function $V(r)$ which gives the volume, in cubic inches, of an ice cream cone that costs \$5 in terms of r only. *Your formula should not include the letter h.*

Solution: The volume of the ice cream cone is given by $\dfrac{\pi r^2 h}{3}$. Using our answer from part a., we have

$$V(r) = \frac{\pi r^2\left(\sqrt{\frac{100}{r^2}-r^2}\right)}{3}.$$

Answer: $V(r) =$ $\dfrac{\pi r^2\left(\sqrt{\frac{100}{r^2}-r^2}\right)}{3}$

c. [3 points] What is the domain of $V(r)$ in the context of this problem?

Solution: Note that r cannot be equal to 0 since then the cost would be 0 rather than \$5, so we know $r > 0$.
Also note that $h^2 \geq 0$. From part a., we know that $h^2 = \frac{100}{r^2} - r^2$, so we have

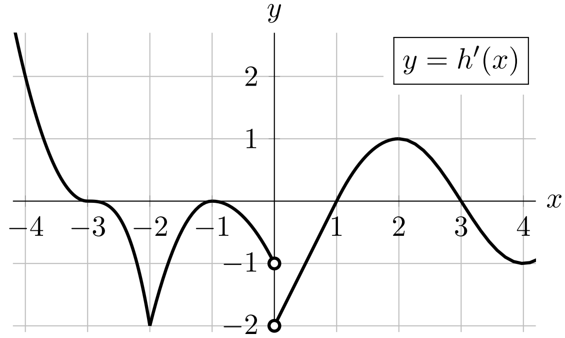
$$\begin{aligned}\frac{100}{r^2}-r^2 &\geq 0\\ 100 &\geq r^4\\ 10 &\geq |r^2| = r^2\\ \sqrt{10} &\geq |r| = r \text{ (since } r>0).\end{aligned}$$

Answer: $(0, \sqrt{10})$ **or** $(0, \sqrt{10}]$

5. [15 points]
Shown on the right is the graph of $h'(x)$, the **derivative** of a function $h(x)$. Assume that h is continuous on its entire domain $(-\infty, \infty)$.

Use this graph to answer the questions below.

You may also use the fact that $\boxed{h(-4) = 5}$.



a. [3 points] Find the linear approximation $L(x)$ of $h(x)$ near $x = -4$, and use your formula to approximate $h(-3.9)$.

Solution:

$$L(x) = h(4) + h'(4)(x - (-4)) = 5 + 2(x + 4)$$

and our linear approximation of $h(-3.9)$ is therefore $h(-3.9) \approx L(-3.9) = 5 + 2(0.1) = 5.2$.

Answer: $L(x) =$ $5 + 2(x + 4)$ and $h(-3.9) \approx$ $5 + 2(0.1) = 5.2$

b. [2 points] Is the estimate of $h(-3.9)$ in part **a.** an overestimate or underestimate of the actual value, or is there not enough information to decide? Briefly explain your reasoning.

Circle one: $\boxed{\text{OVERESTIMATE}}$ UNDERESTIMATE NOT ENOUGH INFORMATION

Brief explanation:

Solution: The second derivative is negative (since $h'(x)$ is decreasing/the slope of $h'(x)$ is negative) on the interval $(-4, -3.9)$ so $h(x)$ is concave down on this interval. Therefore, the tangent line to $y = h(x)$ at $x = -4$ is above the curve $y = h(x)$ at $x = -3.9$ and the resulting linear approximation of $h(-3.9)$ must be an overestimate.

For each question below, circle **all** correct choices. You do not need to justify your answers.

c. [2 points] At which of the following values of x does $h(x)$ have a critical point?

$x = -2$ $\boxed{x = -1}$ $\boxed{x = 0}$ $x = 2$ $\boxed{x = 3}$ NONE OF THESE

d. [2 points] At which of the following values of x does $h(x)$ have a local maximum?

$x = -1$ $x = 0$ $x = 1$ $x = 2$ $\boxed{x = 3}$ NONE OF THESE

e. [2 points] At which of the following values of x does $h(x)$ have an inflection point?

$x = -3$ $\boxed{x = -2}$ $\boxed{x = -1}$ $\boxed{x = 0}$ $\boxed{x = 2}$ NONE OF THESE

f. [2 points] If $g(x) = h'(x)$, on which of the following interval(s) does $g(x)$ satisfy the hypotheses of the Mean Value Theorem?

$[-4, -1]$ $[-1, 2]$ $\boxed{[1, 3]}$ $\boxed{[2, 4]}$ NONE OF THESE

g. [2 points]. If $g(x) = h'(x)$, on which of the following interval(s) does $g(x)$ satisfy the conclusion of the Mean Value Theorem?

$\boxed{[-4, -1]}$ $\boxed{[-1, 2]}$ $\boxed{[1, 3]}$ $\boxed{[2, 4]}$ NONE OF THESE

6. [9 points] The Loads-of-Oats company is designing a new cylindrical container for their steel-cut oats. The company specifies that

- the height of the cylinder and four times the radius of the cylinder should sum to 18 inches
- the radius of the cylinder will be at least 1 inch, and
- the height of the cylinder will be at least 2 inches.

a. [2 points] What is the largest possible radius of such a cylindrical container?

Solution: Let r be the radius of the cylindrical container and h be the height of the cylindrical container. We require $h + 4r = 18$ and $h \geq 2$. Therefore, $18 - 4r \geq 2$ so $16 \geq 4r$ and $4 \geq r$. and so the largest possible radius of such a cylindrical container is 4 inches.

Answer: **4 inches**

b. [7 points] Find the height and radius of such a cylindrical container, in inches, that maximize the volume of the container.

In your solution, make sure to carefully define any variables and functions you use. Use calculus to justify your answers, and show enough evidence that the values you find do in fact maximize the volume.

Solution: As in part a. let r be the radius of the cylindrical container and h be the height of the cylindrical container. The volume of such a cylinder is $\pi r^2 h$. Using the constraint that $h = 18 - 4r$, the volume $V(r)$ of the cylindrical container is given by

$$V(r) = \pi r^2(18 - 4r) = 18\pi r^2 - 4\pi r^3.$$

Because $r \geq 1$ and $r \leq 4$, the domain of $V(r)$ is $[1, 4]$. The derivative of $V(r)$ is

$$V'(r) = 36\pi r - 12\pi r^2.$$

The critical points of $V(r)$ occur when $V'(r) = 0$ or $V'(r)$ does not exist. The latter does not occur so checking when $V'(r) = 0$, we find

$$36\pi r - 12\pi r^2 = 0$$
$$12\pi r(3 - r) = 0$$
$$\text{So } r = 0 \text{ or } r = 3.$$

However, $r = 0$ is not in the domain of the function $V(r)$ because $r = 0$ is not in $[1, 4]$. Therefore, the only critical point of $V(r)$ in this context is $r = 3$. Because $V(r)$ is a continuous function on the closed interval $[1, 4]$, the Extreme Value Theorem (EVT) guarantees a global maximum. So, to find the global maximum, we compare the values of $V(r)$ at the endpoints and critical point.

r	$V(r) = \pi r^2(18 - 4r)$
1	$\pi(18 - 4) = 12\pi$
3	$9\pi(18 - 12) = 54\pi$
4	$16\pi(18 - 16) = 32\pi$

Therefore, $V(r)$ attains its global maximum at $r = 3$. So a radius $r = 3$ inches and a height of $h = 18 - 4(3) = 6$ inches maximizes the volume of the container, and the resulting maximum volume is 54π cubic inches.

Answer: height = **6 inches** and radius = **3 inches**

7. [5 points] A function $g(x)$ is given by the following formula, where K and M are constants:

$$g(x) = \begin{cases} Ke^{-x+5} & x \leq 5 \\ M + \sqrt{x+4} & x > 5. \end{cases}$$

Find all values of K and M so that $g(x)$ is differentiable on $(-\infty, \infty)$. Write NONE if there are no such values. You do not need to simplify your answers, but show your work clearly.

Solution: When $x < 5$, the derivative of $g(x)$ is $g'(x) = -Ke^{-x+5}$. When $x > 5$, the derivative of $g(x)$ is $g'(x) = \dfrac{1}{2\sqrt{x+4}}$. In order for $g(x)$ to be differentiable on $(-\infty, \infty)$, it must be differentiable at $x = 5$. Therefore, we need the slope to the left and right of $x = 5$ to match. That is, we need

$$-Ke^{-5+5} = \frac{1}{2\sqrt{5+4}}.$$

Solving for K, we have

$$K = -\frac{1}{2(3)} = -\frac{1}{6}.$$

Requiring that $g(x)$ is differentiable at $x = 5$ also requires that $g(x)$ be continuous at $x = 5$. Using this and the value of K we found above to solve for M, we have

$$\begin{aligned} Ke^{-5+5} &= M + \sqrt{5+4} \\ -\frac{1}{6} &= M + 3 \\ M &= -\frac{1}{6} - 3 \\ M &= -\frac{19}{6}. \end{aligned}$$

Answer: $K =$ $-\dfrac{1}{6}$ and $M =$ $-\dfrac{19}{6}$

8. [8 points] Prairie dogs named Paws and Dot have been hard at work digging a tunnel. Consider the functions L and C defined as follows:

- $L(w)$ is the length of the tunnel, in feet, when w pounds of dirt have been removed.
- $C(w)$ is the total number of Calories the prairie dogs have burned digging their tunnel when they have removed a total of w pounds of dirt for their tunnel.

The functions $L(w)$ and $C(w)$ are both invertible and differentiable.

a. [4 points] Complete the sentence below to give a practical interpretation of the equation

$$(L^{-1})'(10) = 24.$$

In order to increase the length of the tunnel from 10 *feet to* 10.25 *feet, ...*

Solution: the prairie dogs have to remove approximately 6 pounds of dirt.

b. [4 points]

i. Which of the following expressions gives the length, in feet, of the prairie dog tunnel when the prairie dogs have burned a total of x Calories digging? Circle the one correct expression.

$C(L^{-1}(x))$ $\quad$ $C^{-1}(L(x))$ $\quad$ $\boxed{L(C^{-1}(x))}$ $\quad$ $L^{-1}(C(x))$

ii. Use the answer you selected in part i to find an expression for the instantaneous rate of change of the length of the prairie dog tunnel, in feet per calorie, when the prairie dogs have burned a total of 2000 calories digging.
Simplify as much as possible. Note that your final answer may involve the function names L, L^{-1}, L' , C, C^{-1}, *and* C' *but should not involve the function names* $\left(L^{-1}\right)'$ *or* $\left(C^{-1}\right)'$.

Solution: We are trying to find a simplified expression for $\frac{d}{dx}\left(L\left(C^{-1}(x)\right)\right)$ at $x = 2000$. Using the chain rule and the formula for the derivative of the inverse of a function, we have

$$\begin{aligned} \frac{d}{dx}\left(L\left(C^{-1}(x)\right)\right) &= L'\left(C^{-1}(x)\right) \cdot \left(C^{-1}\right)'(x) \\ &= L'\left(C^{-1}(x)\right) \cdot \frac{1}{C'\left(C^{-1}(x)\right)}. \end{aligned}$$

So at $x = 2000$, we have

$$L'\left(C^{-1}(2000)\right) \cdot \frac{1}{C'\left(C^{-1}(2000)\right)} = \frac{L'\left(C^{-1}(2000)\right)}{C'\left(C^{-1}(2000)\right)}.$$

Answer: $L'\left(C^{-1}(2000)\right) \cdot \frac{1}{C'\left(C^{-1}(2000)\right)} = \frac{L'\left(C^{-1}(2000)\right)}{C'\left(C^{-1}(2000)\right)}$

9. [11 points] A continuous function $w(x)$ and its derivative $w'(x)$ are given by

$$w(x) = \begin{cases} x^2(3x^2+10x-9) & x \le 1 \\ -2\ln(3x-2)+4 & x > 1 \end{cases} \qquad \text{and} \qquad w'(x) = \begin{cases} 6x(x+3)(2x-1) & x < 1 \\ \dfrac{-6}{3x-2} & x > 1. \end{cases}$$

a. [2 points] Find the x-coordinates of all critical points of $w(x)$. If there are none, write NONE. You do not need to justify your answer.

Solution: The critical points of $w(x)$ occur when $w'(x) = 0$ or $w'(x)$ does not exist. Looking at the formula provided for the derivative, $w'(x) = 0$ when $x = 0, -3$, and $\frac{1}{2}$ and $w'(x)$ does not exist when $x = 1$*. Note that although the denominator of the second piece of w' is 0 when $x = 2/3$, the formula for $w'(x)$ when $x = 2/3$ is given by the first piece. So this is not a critical point.
(*We can also verify that $w'(1)$ is not defined by comparing the values of the first and second pieces of w' at 1.)

Answer: Critical point(s) at $x =$ $-3, 0, \dfrac{1}{2}, 1$

For each part below, you must use calculus to find and justify your answers. Make sure your final conclusions are clear, and that you show enough evidence to justify those conclusions.

b. [4 points] Find the x-coordinates of all global minima and global maxima of $w(x)$ **on the interval** $(-\infty, 0)$. If there are none of a particular type, write NONE.

Solution: The only critical point of $w(x)$ on $(-\infty, 0)$ is $x = -3$. We have

- $w(-3) = (-3)^2(3(-3)^2 + 10(-3) - 9) = 9(27 - 30 - 9) = 9(-12) = -108$
- $\lim_{x \to -\infty} w(x) = \infty$
- $\lim_{x \to 0^-} w(x) = 0.$

Therefore, there is a global minimum at $x = -3$ and no global maximum.

Answer: Global min(s) at $x =$ -3 and Global max(es) at $x =$ **None**

c. [5 points] Find the x-coordinates of all global minima and global maxima of $w(x)$ **on the interval** $\left[-1, \frac{e+2}{3}\right]$. If there are none of a particular type, write NONE.
In case it is useful, note that $1 < \frac{e+2}{3} < 2$.

Solution: The critical points in $\left[-1, \frac{e+2}{3}\right]$ are $x = 0$, $x = \frac{1}{2}$, $x = 1$.
Note that the Extreme Value Theorem applies here, so we are guaranteed both a global minimum and global maximum. Comparing values of w at critical points and endpoints, we have

- $w(-1) = (-1)^2(3(-1)^2 + 10(-1) - 9) = 3 - 10 - 9 = -16$
- $w(\frac{e+2}{3}) = -2\ln(e + 2 - 2) + 4 = -2\ln(e) + 4 = -2 + 4 = 2$
- $w(0) = 0^2(3(0)^2 + 10(0) - 9) = 0$
- $w(\frac{1}{2}) = (\frac{1}{2})^2(3(\frac{1}{2})^2 + 10(\frac{1}{2}) - 9) = \frac{1}{4}(\frac{3}{4} + 5 - 9) = \frac{1}{4}(\frac{3}{4} - 4) = \frac{3}{16} - 1 = -\frac{13}{16}$
- $w(1) = (1)^2(3(1)^2 + 10(1) - 9) = 3 + 10 - 9 = 4.$

Therefore, there is a global minimum at $x = -1$ and a global maximum at $x = 1$.

Answer: Global min(s) at $x =$ -1 and Global max(es) at $x =$ 1

10. [8 points] Some information about the derivative $p'(x)$ and the second derivative $p''(x)$ of a function $p(x)$ is provided in the table below.

x	-4	-3	-2	-1	0	1	2
$p'(x)$	1	0	-2	0	-1	0	2
$p''(x)$	-1	0	0	0	0	2	1

Assume that

- $p''(x)$ is defined and continuous on the interval $(-\infty, \infty)$ and
- the values of both $p'(x)$ and $p''(x)$ are strictly positive or strictly negative between consecutive table entries.

For each question below, circle **all** correct choices. You do not need to justify your answers.

a. [2 points] On which of the following intervals must $p(x)$ be always concave up?

$-4 < x < -3$ $\quad$ $-3 < x < -2$ $\quad$ $\boxed{-2 < x < -1}$

$-1 < x < 0$ $\quad$ $\boxed{0 < x < 1}$ $\quad$ $\boxed{1 < x < 2}$ $\quad$ NONE OF THESE

b. [2 points] At which of the following values of x must $p(x)$ have a local minimum?

$x = -3$ $\quad$ $x = -2$ $\quad$ $x = -1$ $\quad$ $x = 0$ $\quad$ $\boxed{x = 1}$ $\quad$ NONE OF THESE

c. [2 points] At which of the following values of x must $p(x)$ have an inflection point?

$x = -3$ $\quad$ $\boxed{x = -2}$ $\quad$ $\boxed{x = -1}$ $\quad$ $\boxed{x = 0}$ $\quad$ $x = 1$ $\quad$ NONE OF THESE

d. [2 points] At which value(s) of x does $p(x)$ attain a global maximum on the interval $[-4, 0]$?

$x = -4$ $\quad$ $\boxed{x = -3}$ $\quad$ $x = -2$ $\quad$ $x = -1$ $\quad$ $x = 0$

NONE OF THESE $\quad$ CANNOT BE DETERMINED