6. (13 pts) As part of your new job as Assistant Inventor at "All Things Foam" you have invented a product you call the "Superfoamy Supercomfy Wavy Mattress," and you have delivered the following specifications to Asha, the factory manager:

(Cross Section of the Superfoamy Supercomfy Wavy Mattress)

"I'm sorry," says Asha. "Regulations demand that all new product schematics be given also in formula form."

a) Satisfy the factory regulations by writing a formula for the height \( H \) of the mattress, as a function of the horizontal distance \( d \) from the head of the mattress.

We will use a cosine function to describe the "wavy profile." We have to adjust the frequency and amplitude to get a correct fit to the picture. We want 3.6 bumps altogether in 72 inches (or six feet), or one bump every two inches. \( \cos \left( \frac{2\pi}{2} \cdot \frac{d}{72} \right) \).

will have one peak every 2 units (inches) in \( d \).

The amplitude of the wave profile will be \( \frac{1}{2} \times 1.4 \) in. = 0.7 inches, and the wave oscillates around height 1.8 in + 0.7 in = 2.5 in. Altogether, the profile is described by \( h(x) = 2.5 + 0.7 \cos \left( \frac{2\pi}{72} x \right) \).

b) What are the domain and range of your function?

The domain is \([0, 72]\) (in units of inches), and the range is \([1.8, 3.2]\).