7. (15 pts) Your local cable internet provider has discovered that the strength (measured in Watts) of the signal generated at their broadcast station decreases fairly rapidly as it travels over their wires. They are concerned about this because subscribers must receive at least a 12-Watt signal in order for their systems to work. Engineers have calculated that $S = f(d) = 160(0.64)^d$, where S is the signal strength, and d represents distance (in miles) from the broadcast station.

a) Translate the statement "$f^{-1}(6) = 11$" into plain English. Is this statement true or false?

The distance from the source where the signal strength is 6 Watts is 11 miles. Not true if $f(d) = 160(0.64)^d$. For $d = 11$, $f(11)$ is much smaller than 6.

b) What percentage of the signal is lost over each mile of cable?

For each gain of 1 in d, the signal strength received is multiplied by 0.64. Thus, a fraction 0.36, or 36 %, is lost each mile.

c) What does the "160" tell you, in real world terms? (answer in a complete sentence.)

The initial signal is 160 Watts strong.

d) How far from the station can one live and still receive the service?

Since $f(d)$ is a decreasing function (exponential function with base $0.64 < 1$), we just solve for d in: $12 = f(d) = 160(0.64)^d$

But then $(0.64)^d = \frac{3}{40}$, $d = \ln(3/40) / \ln(0.64) = 5.804$

e) Better wiring is installed which cuts in half the percentage of the signal lost per mile. You find that the signal strength at your house is doubled! How far from the station do you live?

When this happens, the new function for S is

$g(d) = 160\,(0.82)^d$.

We want to solve $g(d) = 2f(d)$, or

$(0.82)^d = 2\,(0.64)^d$.

Take logs, and solve for d: $d = \dfrac{\ln 2}{\ln(0.82/0.64)} = 2.797.$