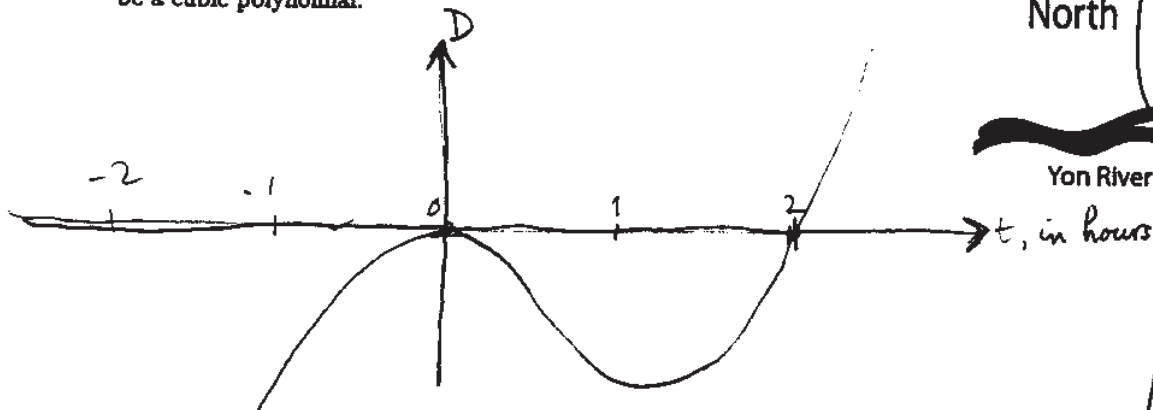


8. (13 pts) Johnny Howard, the cubical long-nosed echidna, makes a habit of travelling so that his displacement from Yon River (as a function of time) is always a third degree (cubic) polynomial. This morning he left his home, travelling north to take a basket of scones to his Aunt Hillary. At 12:00 noon, he reached Yon River, but discovered he had forgotten the jam. He then went home again for jam, then back to the river. He crossed the river at 2:00 pm and proceeded to Hillary's house.

Let  $t$  be the time in hours after noon (so morning = negative  $t$ ), and let  $D$  be Johnny's displacement north of the river in kilometers (south = negative displacement).

a) Sketch a graph of  $D$  against  $t$ , keeping in mind that the function must be a cubic polynomial.



b) Write a possible formula for  $D$  as a function of  $t$ .

$$D(t) = t^2 (t-2)$$

This is a cubic polynomial which crosses the  $t$ -axis at  $t=0$  and  $t=2$ , and has  $D'(0) = 0$  as well.

c) Now modify your formula to include some additional information: At 3:00 pm, Johnny was 2 kilometers north of the river.

Figure 2: Map of the region

Correct the  $D(t)$  in part (b) as follows:

$D(t) = c t^2 (t-2)$ , where  $c$  is a positive constant, will also match the graph above.

To fit the <sup>(new)</sup> condition,  $D(3) = 2$  requires

that  $D(3) = c 3^2 (3-2) = 9c = 2$ , or

$c = \frac{2}{9}$ . Thus,  $D(t) = \frac{2}{9} t^2 (t-2)$  will fit the pattern of the graph and the new condition.