9. (15 pts) The tortoise, the hare, and the rhinoceros begin a 9-mile race at $t = 0$ hours. Remarkably, a 3-way tie results – it takes each competitor exactly 2 hours to finish.

The tortoise's style is slow and steady. He runs the entire race without speeding up or slowing down at all. The hare's style is more erratic: He runs half of the race in the first 20 minutes, stops for a long tea, then runs the second half in the last 20 minutes. The rhino, an amateur mathematician, runs so that her position $R(t)$ in miles from the starting line is always exactly $4.5t^{3-t}$.

a) What is the average velocity on the time-interval $[0, 2]$ of...

i) ...the tortoise?

The average velocity is $\frac{9 \text{ mi}}{2 \text{ hrs}} = 4.5 \text{ mi/hr}$

ii) ...the hare?

The same average velocity, 4.5 mi/hr

iii) ...the rhinoceros?

The same average velocity, 4.5 mi/hr

b) What is the instantaneous velocity of the tortoise at time $t = 1$?

The tortoise moves with constant velocity, so his instantaneous velocity at $t = 2$ is 4.5 mi/hr.

c) What is the instantaneous velocity of the hare at time $t = 1$?

The hare is stationary from $t = 1/3$ hr. to $t = 1\,2/3$ hr, so at $t = 1$, its velocity is 0.

d) Estimate the instantaneous velocity of the rhinoceros at time $t = 1$. (Show your work. "I used my calculator" is *not* sufficient work.)

The rhino is at 4.5 mi. at $t = 1$. To estimate the velocity at $t = 1$, compute the average velocity from $t = 1$ to $t = 1.1$. This is $\dfrac{4.5\,(1.1)^{3-1.1} - 4.5\,(1)^2}{0.1}$

≈ 8.934 (By calculator).

e) Imagine that you are a radio reporter describing the events as you see them at time $t = 1$. Tell your audience the status of the race. For example, is anyone passing anyone else?

At $t = 1$, all three are tied at 4.5 mi. The hare is still, the tortoise is moving at speed 4.5 mi/hr and the rhino at speed approximately 8.934 mi/hr. Unless something changes it looks as though the rhino will win. (Something does change.)