5. (8 points) (a) Find a value of k so that the function

$$f(x) = \begin{cases} 1 - x, & \text{if } x < 3; \\ kx - 4k, & \text{if } x \geq 3. \end{cases}$$

is continuous on every interval.

The function is continuous for all $x \neq 3$ since both functions are linear. We need them to meet at $x = 3$. Thus,

$1 - 3 = k(3) - 4k \rightarrow -2 = -k$, so $k = 2$

(b) Is the function you found differentiable at $x = 3$? Explain why or why not.

As $x \to 3^-$, the slope of f is -1, but as $x \to 3^+$, the slope is 2 (since for $x \geq 3$, $f(x) = 2x - 8$). There is a sharp corner @ $x = 3$. Thus, f is not differentiable at $x = 3$. There is a corner in the graph

6. (12 points) Are the given statements true or false? Give an explanation for each answer.

(a) If the graph of a function g is obtained by shifting the graph of a function f vertically upward by 3 units, then $g' = f' + 3$.

False. If $g(x) = f(x) + 3$, then $g'(x) = f'(x)$. A vertical shift does not change the slope of f.

(b) If a function is not differentiable then it is not continuous.

False. For example, $y = |x|$ is continuous at $x = 0$ but not differentiable there.

(c) If $f'' > 0$, then f is increasing.

False. For example, $y = e^{-x}$ is decreasing, but $f'' > 0$.

(d) The inequality $\sqrt{x} < 2\log(x^4)$ holds for large positive values of x (that is, as $x \to +\infty$).

False. As $x \to \infty$, $\sqrt{x}$ (or any positive power of x) will outtake the log function -- no matter what the co-efficient of the log (or power of x ...).