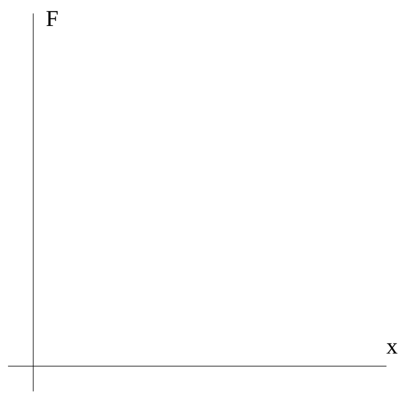
(10.) (12 points) When you weigh yourself by standing on a bathroom scale, you push down on a spring inside the scale. As the spring compresses – that is, as it decreases in length – your body is acted on by two forces:

- Gravity exerts a downward force $F_g$ on your body. The magnitude of this force is $mg$, where $m$ is the mass of your body, and $g$ is a constant.
- The spring in the scale exerts an upward force $F_s$ on your body. The magnitude of this force is directly proportional to the total change in the spring's length. The constant of proportionality $k$ is called the *spring constant* of the spring.

The net downward force $F$ on your body equals the difference $F_g - F_s$.

(a) Write an expression for $F$ as a function of $x$, the length by which the spring has been compressed.

(b) On the axes below, sketch a graph of $F$ as a function of $x$, clearly labelling both intercepts.




(c) What is the significance of the $x$-intercept of this graph? Hint: we will refer to this $x$-value as $x_{\text{eq}}$.

(d) The mechanism inside the scale doesn't actually measure your mass $m$ directly; instead, it measures the value of $x_{\text{eq}}$. However, it turns out that $m$ and $x_{\text{eq}}$ only differ by multiplication by a constant factor – that is, $m = c \cdot x_{\text{eq}}$, for some $c$. This means that the numbering of the scale's display can be chosen so that the scale gives a readout of your mass, after all.

What is the value of the constant $c$?