(10.) (12 points) When you weigh yourself by standing on a bathroom scale, you push down on a spring inside the scale. As the spring compresses – that is, as it decreases in length – your body is acted on by two forces:

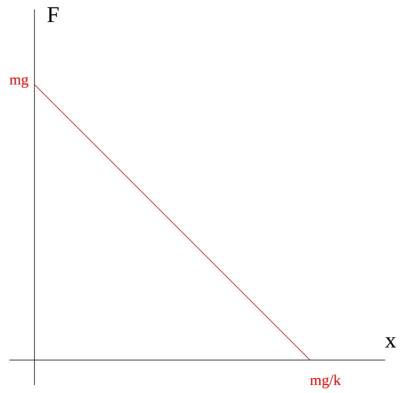
- Gravity exerts a downward force F_g on your body. The magnitude of this force is mg, where m is the mass of your body, and g is a constant.
- The spring in the scale exerts an upward force F_s on your body. The magnitude of this force is directly proportional to the total change in the spring's length. The constant of proportionality k is called the *spring constant* of the spring.

The net downward force F on your body equals the difference $F_g - F_s$.

(a) Write an expression for F as a function of x, the length by which the spring has been compressed.

$$F = mg - kx$$

(b) On the axes below, sketch a graph of F as a function of x, clearly labelling both intercepts.



(c) What is the significance of the x-intercept of this graph? Hint: we will refer to this x-value as x_{eq}.

When $x = x_{\text{eq}}$, the net force F is zero, because the gravitational force and the force from the spring are in *equilibrium*. Since there is no net force on your body, you stop sinking down into the scale.

(d) The mechanism inside the scale doesn't actually measure your mass m directly; instead, it measures the value of x_{eq}. However, it turns out that m and x_{eq} only differ by multiplication by a constant factor – that is, $m = c \cdot x_{\text{eq}}$, for some c. This means that the numbering of the scale's display can be chosen so that the scale gives a readout of your mass, after all.

What is the value of the constant c?

$$x_{\text{eq}} = \frac{mg}{k}, m = \left(\frac{k}{g}\right) x_{\text{eq}}$$

$$\boxed{c = \frac{k}{g}}$$