(6.) (6 points) Let  $f(x) = x^{3x}$ . Use the **definition** of the derivative to express f'(2) as a limit. You do not need to simplify your expression or try to estimate f'(2).

$$f'(2) = \lim_{h \to 0} \frac{(2+h)^{3(2+h)} - 2^6}{h}$$

(7.) (8 points) Suppose g is a differentiable function that satisfies the following three properties:

- 1. g is concave up.
- 2. g(1) = 9.
- 3. g(5) = 3.
- (a) What is the average rate of change of g on the interval [1, 5]?

$$\frac{3-9}{5-1} = -\frac{6}{4} = -\frac{3}{2}$$

(b) Which is larger, g'(2) or g'(4)? Explain.

Since g is concave up, we know that g'' > 0. This means that g' is increasing, so g'(4) > g'(2).

(c) What is the maximum possible value for g(3)? (Hint: try sketching a graph of g.) Explain your reasoning.

A sketch suggests the key idea: since g is concave up, the graph of g between x = 1 and x = 5 must be *lower* than the secant line connecting the points (1, 9) and (5, 3). This line passes through the point (3, 6), and so it must be the case that g(3) < 6.