(6.) (6 points) Let \( f(x) = x^{3x} \). Use the definition of the derivative to express \( f'(2) \) as a limit. You do not need to simplify your expression or try to estimate \( f'(2) \).

\[
f'(2) = \lim_{h \to 0} \frac{(2+h)^{3(2+h)} - 2^6}{h}
\]

(7.) (8 points) Suppose \( g \) is a differentiable function that satisfies the following three properties:

1. \( g \) is concave up.
2. \( g(1) = 9 \).
3. \( g(5) = 3 \).

(a) What is the average rate of change of \( g \) on the interval \([1, 5]\)?

\[
\frac{3-9}{5-1} = \frac{-6}{4} = -\frac{3}{2}
\]

(b) Which is larger, \( g'(2) \) or \( g'(4) \)? Explain.

Since \( g \) is concave up, we know that \( g'' > 0 \). This means that \( g' \) is increasing, so \( g'(4) > g'(2) \).

(c) What is the maximum possible value for \( g(3) \)? (Hint: try sketching a graph of \( g \).) Explain your reasoning.

A sketch suggests the key idea: since \( g \) is concave up, the graph of \( g \) between \( x = 1 \) and \( x = 5 \) must be lower than the secant line connecting the points \((1, 9)\) and \((5, 3)\). This line passes through the point \((3, 6)\), and so it must be the case that \( g(3) < 6 \).