(6.) (6 points) Let $f(x) = x^{3x}$. Use the **definition** of the derivative to express $f'(2)$ as a limit. You do not need to simplify your expression or try to estimate $f'(2)$.

$$f'(2) = \lim_{h \to 0} \frac{(2+h)^{3(2+h)} - 2^6}{h}$$

(7.) (8 points) Suppose g is a differentiable function that satisfies the following three properties:

1. g is concave up.
2. $g(1) = 9$.
3. $g(5) = 3$.

(a) What is the average rate of change of g on the interval $[1, 5]$?

$$\frac{3-9}{5-1} = -\frac{6}{4} = \boxed{-\frac{3}{2}}$$

(b) Which is larger, $g'(2)$ or $g'(4)$? Explain.

Since g is concave up, we know that $g'' > 0$. This means that g' is increasing, so $g'(4) > g'(2)$.

(c) What is the maximum possible value for $g(3)$? (Hint: try sketching a graph of g.) Explain your reasoning.

A sketch suggests the key idea: since g is concave up, the graph of g between $x = 1$ and $x = 5$ must be *lower* than the secant line connecting the points $(1, 9)$ and $(5, 3)$. This line passes through the point $(3, 6)$, and so it must be the case that $g(3) < 6$.