10. (13 points) The traffic on US-23 between Brighton and Ann Arbor is stop and go every weekday morning. I merge onto US-23 South at Brighton travelling 35 miles per hour. The traffic is bad and I must immediately slow down, finally coming to a stop 2 miles after I got on the highway. I am able to speed right up again, and I reach my maximum speed of 70 miles per hour six miles after I merged onto US-23. There are again traffic problems and I must slow again, coming to a stop 4 miles after I reached my peak speed. Suppose my speed continues in the same pattern until I reach the Ann Arbor exit, 13 miles after I merged onto the highway at Brighton.

(a) Assume that my speed may be modelled by a trigonometric function and sketch a graph of my speed as I travel south on US-23. Let the horizontal axis represent my distance from the Brighton entrance to the highway. Be sure to appropriately label the axes.



(b) Determine a trigonometric function, v, giving my speed as a function of d, my distance from Brighton.

$$v(d) = -35\sin(\frac{\pi}{4}d) + 35$$

(c) What was my speed when I reached Ann Arbor?

To find the speed as I reach Ann Arbor, one merely needs to plug d = 13 into v(d), obtaining 59.75 miles per hour.

(d) What are the units of v'?

First note that v is a function of d, not time which makes this a little trickier. So we are really looking at $v'(d) = \frac{dv}{dd}$. Then one sees that the units are (miles/hour)/miles = 1/hour.