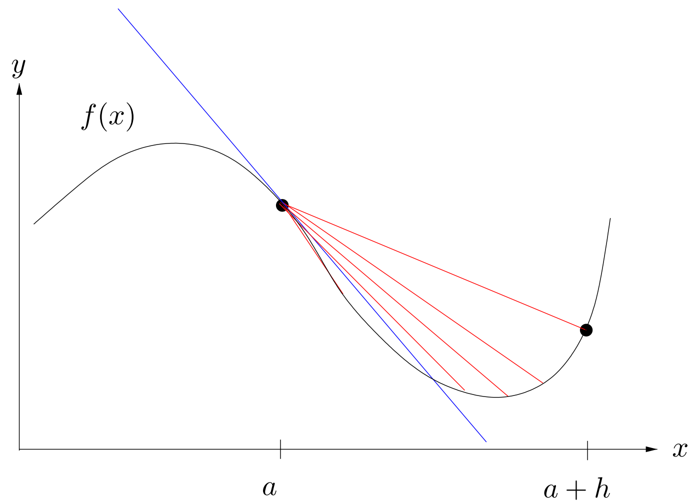
7. (10 points) For this problem f is differentiable everywhere.

(a) Write the limit definition of the derivative of the function f at the point a.

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

(b) On the graph below, show how the average rate of change of f between $x = a$ and $x = a + h$ is related to the derivative at the point a. Give a brief explanation of your illustration including how the limit as $h \to 0$ is demonstrated in your picture.



The average rate of change of f between $x = a$ and $x = a+h$ is the slope of the red line connecting $f(a)$ and $f(a+h)$. This is an approximation to the derivative of f at $x = a$, which is the slope of the tangent line (shown in blue) of f at $x = a$. One can see as the h gets smaller ($h \to 0$), the slopes of the red lines become better and better approximations to the slope of the tangent line to f and $x = a$.

(c) Write the limit definition for $f'(2)$ if $f(x) = e^{\sin 2x}$. [You do not need to find the limit or approximate $f'(2)$.]

$$f'(2) = \lim_{h \to 0} \frac{e^{\sin 2(2+h)} - e^{\sin 4}}{h}$$