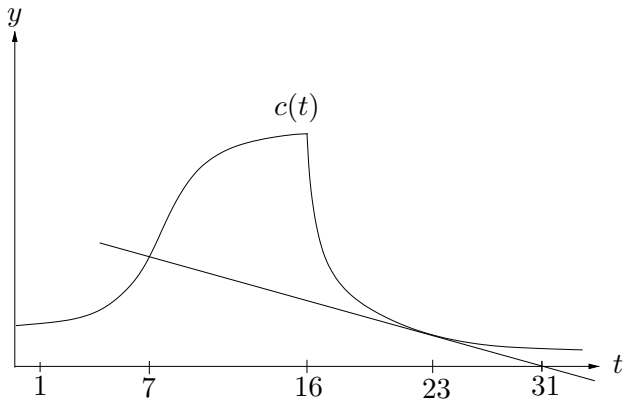
8. (14 points) As Sweetest Day (October 16th) approaches, millions of Americans flock to stores to buy their special someone a card. The number of cards sold can be approximated by the continuous function c graphed below where $c(t)$ gives the number of cards sold on day t and $t = 1$ corresponds to October 1.



(a) Are there any t values where the function may not be differentiable? Explain.

The function c is not differentiable at $t = 16$ as the function has a sharp corner here (or a vertical tangent–either will do).

(b) Explain the concavity of the graph between October 1 and October 16 in the context of this problem.

As the days pass after October 1st, people are flocking to the stores at an increasing rate until approximately October 8th. So more cards are being sold each successive day between October 1st and the 8th. At this point, the number of people still needing to buy cards slows and the rate is decreasing until to approximately 0 on Sweetest day, when there are nearly the same number of cards sold as on the day before.

(c) If on October 1 there are 30,000 cards sold and on October 23 there are 25,000 cards sold, what is the average rate of change of $c(t)$ over this time? Express your final answer in sentence form and in the context of this problem.

The average rate of change of $c(t)$ over the interval $t = 1$ to $t = 23$ is given by the expression

$$\frac{c(23) = c(1)}{23 - 1} = \frac{25,000 - 30,000}{22} = \frac{-5,000}{22} \approx -227.$$

This means that between October 1st and October 23 the number of cards decreases by approximately 227 cards each day.

(d) How many cards were sold on October 7? Show your work.

One needs to use the points $(23, 25,000)$ and $(31, 0)$ to find the equation of the pictured line. The slope of the line is $m = \frac{0-25,000}{31-23} = -3125$. So the equation of the line is $y = -3125(t - 31)$. To find how many cards were sold on October 7th we only need to plug $t = 7$ in, which gives 75,000. Therefore, on October 7th there were 75,000 cards sold.