9. (5 points) Write the **limit definition** for the derivative of $e^{\sin(x)}$ with respect to x. (No need to simplify or to attempt to find the limit.)

$$\lim_{h \to 0} \frac{e^{\sin(x+h)} - e^{\sin(x)}}{h}$$

10. (9 points) Suppose

$$f(x) = \begin{cases} e^{\sin(x)} & x < \frac{\pi}{2} \\ kx & x \geq \frac{\pi}{2} \end{cases}$$

where k is some constant.

(a) If f is continuous, what is the value of k?

To ensure continuity, the two branches must have the same value at $x = \frac{\pi}{2}$. Therefore, $e^{\sin(\pi/2)} = k\frac{\pi}{2}$ is forced. Solving gives $k = \frac{2e}{\pi}$.

(b) Compute the average rate of change of f between $x = 1.5$ and $x = \frac{\pi}{2}$.

The number we want is

$$\frac{\Delta f}{\Delta x} = \frac{e - e^{\sin(1.5)}}{\pi/2 - 1.5} = .09606$$

(c) Compute the average rate of change of f between $x = 1.57$ and $x = \frac{\pi}{2}$.

The number we want is

$$\frac{\Delta f}{\Delta x} = \frac{e - e^{\sin(1.57)}}{\pi/2 - 1.57} = .00108$$

(d) Do you think f is differentiable at $x = \frac{\pi}{2}$? Explain your answer. [Your work from parts (a) - (c) may be useful here.]

The function will not be differentiable. The rate of change of f at $\frac{\pi}{2}$ approaches 0 from the left, but from the right it is $k = \frac{2e}{\pi} \neq 0$. Therefore, the function has a cusp or sharp corner at $x = \frac{\pi}{2}$.