9. (5 points) Write the limit definition for the derivative of \( e^{\sin(x)} \) with respect to \( x \). (No need to simplify or to attempt to find the limit.)

\[
\lim_{h \to 0} \frac{e^{\sin(x+h)} - e^{\sin(x)}}{h}
\]

10. (9 points) Suppose

\[
f(x) = \begin{cases} 
e^{\sin(x)} & x < \frac{\pi}{2} \\
kx & x \geq \frac{\pi}{2}
\end{cases}
\]

where \( k \) is some constant.

(a) If \( f \) is continuous, what is the value of \( k \)?

To ensure continuity, the two branches must have the same value at \( x = \frac{\pi}{2} \). Therefore, \( e^{\sin(\pi/2)} = k \frac{\pi}{2} \) is forced. Solving gives \( k = \frac{2e}{\pi} \).

(b) Compute the average rate of change of \( f \) between \( x = 1.5 \) and \( x = \frac{\pi}{2} \).

The number we want is

\[
\frac{\Delta f}{\Delta x} = \frac{e - e^{\sin(1.5)}}{\pi/2 - 1.5} = .09606
\]

(c) Compute the average rate of change of \( f \) between \( x = 1.57 \) and \( x = \frac{\pi}{2} \).

The number we want is

\[
\frac{\Delta f}{\Delta x} = \frac{e - e^{\sin(1.57)}}{\pi/2 - 1.57} = .00108
\]

(d) Do you think \( f \) is differentiable at \( x = \frac{\pi}{2} \)? Explain your answer. [Your work from parts (a) - (c) may be useful here.]

The function will not be differentiable. The rate of change of \( f \) at \( \frac{\pi}{2} \) approaches 0 from the left, but from the right it is \( k = \frac{2e}{\pi} \neq 0 \). Therefore, the function has a cusp or sharp corner at \( x = \frac{\pi}{2} \).