4. (12 points) The cost of gasoline has risen dramatically in the last six months. At the beginning of March, the cost of gasoline was \$2.10 per gallon, but at the beginning of September, the cost was \$3.00 per gallon.

(a) Suppose the cost of gasoline, C, measured in dollars per gallon is a linear function of time, t. Find a formula for cost of gasoline as a function t, in months, since the beginning of March.

Since $t = 0$ represents March, we have the vertical intercept of 2.10. The slope can be found by

$$\frac{\Delta C}{\Delta t} = \frac{3.00 - 2.10}{6} = \frac{.90}{6} = 0.15$$

Thus, $C(t) = 0.15t + 2.10$.

(b) Suppose further that you drive 200 miles per month and that your car averages 27 miles per gallon. Use your formula from part (a) to calculate the price of gasoline at the beginning of December. Assuming that the cost stays the same throughout the month of December, calculate your gas cost for the month of December.

In December, $t = 9$, so using our formula from (a), the cost of gas at the beginning of December will be $C(9) = \$3.45$ per gallon. We will use $\frac{200}{27} = 7.41$ gallons. Therefore, our total cost will be

$$7.41 \times \$3.45 = \$25.56$$

(c) Now suppose instead that C is an exponential function of time. Find a formula for the cost of gas as a function of t, in months since March.

The form of our function is:

$$C(t) = C_0 B^t$$

with $C_0 = 2.10$. Using the point $(6, 3.00)$, we have

$$3 = 2.10(B)^6$$

which gives $B = 1.0612$. Thus, if the increase is exponential, $C(t) = 2.10(1.0612)^t$.

(d) Use your formula from part (c) to calculate the cost of gasoline at the beginning of December. Assuming that this cost remains the same throughout December, use the milage information from part (b) to calculate your total gasoline cost for December.

Using the formula from (c) we find that the cost of gasoline at the beginning of December is $C(9) = \$3.58$ per gallon. Therefore, the total cost is

$$7.41 \times \$3.58 = \$26.56$$