5. (12 points) In Ann Arbor the earliest sunset is at 4 p.m. and the latest at 8 p.m. (ignoring daylight savings time).

(a) Determine a trigonometric function, f, as a function of t in days, where $f(t)$ gives the number of hours past midnight when sunset occurs. Assume that $t = 0$ represents the winter solstice (December 21) and ignore leap years. [Recall that winter solstice is the shortest day of each year.]

The period is clearly 365 days so that $B = \frac{2\pi}{365}$. The minimum and maximum values are 16 and 20, respectively so that $|A| = 2$ and $k = 18$ is the midline. Therefore, f can be given by:

$$f(t) = -2\cos(\frac{2\pi}{365}t) + 18$$

(b) Give a practical interpretation of $f(90)$ in the context of this problem.

The expression $f(90)$ represents the time in hours after midnight that the sun will sun will set 90 days after the winter solstice.

(c) Interpret $f'(120) = 0.03$ in the context of this problem.

The sun will set approximately 0.03 hours (1.8 minutes) later on the 121st day after the winter solstice than on the 120th day after the winter solstice.

(d) Suppose $g(x) = cf(x + h) - k$ for positive constants c, h and k. Give the following for $g(x)$ (your answers may involve c, h and k):

We can evaluate $cf(x + h) - k$ directly, using our formula from (a). We get:

$$g(x) = -2c\cos(\frac{2\pi}{365}(t + h)) + 18c - k$$

. Using this expression we can read off the answers to the following questions below.

(i) Amplitude 2c

(ii) Midline 18c-k

(iii) Period 365