6. (12 points) Consider the function $f(x) = \sin(x^2)$.

   (a) Explain what the difference quotient $\dfrac{\sin(\sqrt{\pi}^2) - \sin(0)}{\sqrt{\pi}}$ represents.

   (b) Write the **limit definition** for $f'(\sqrt{\pi})$ without using the symbol f. No need to numerically evaluate the limit or approximate $f'(\sqrt{\pi})$.

   (c) Suppose that g is a new function defined as follows:

   $$g(x) = \begin{cases} 2f(x) & x < \sqrt{\pi/2} \\ kx + 4 & x \geq \sqrt{\pi/2} \end{cases} \quad \text{for } f(x) \text{ as above}$$

   For what value of k is the function g continuous?