

4. In 1950, a very invasive tree was introduced into a mountainous region of South Africa. Since then, the number of trees has grown exponentially. Suppose that  $N(t)$  gives the number of trees, in *thousands*, as a function of time,  $t$ , measured in years since 1950.

(a) (3 points) In the context of this problem, explain the meaning of  $N^{-1}(25) = 46$ .

The equation  $N^{-1}(25) = 46$  means that in 1996 there were 25,000 of the invasive trees in the region.

(b) (3 points) In the context of this problem, explain the meaning of  $N'(46) = 3$ .

In practical terms,  $N'(46) = 3$  indicates that between 1996 and 1997, the number of invasive trees would increase by approximately 3000 trees.

(c) (4 points) If 100 trees were introduced in 1950, use the above information to find a formula for  $N(t)$ .

The equation will be of the form

$$N(t) = N_o a^t$$

where  $N_o = 0.1$  and  $N(t) = 25$  when  $t = 46$ .

Thus,

$$\begin{aligned} 25 &= 0.1a^{46}, \\ \text{so, } 250 &= a^{46}, \end{aligned}$$

which gives

$$a = 250^{1/46} = 1.1275.$$

An equation for  $N(t)$  is then

$$N(t) = 0.1(1.1275)^t.$$