

8. Astronauts travel to the moon and perform an experiment where they launch a special ball in the air. The ball is able to record its height above the surface of the moon at one second intervals, but when the ball lands it is damaged and the only information that the astronauts can recover is given in the following table:

time (seconds)	0	5	12	18	20	25	30	36	42	60
height (meters)	40	309.25	616.48	815.08	868	971.25	1033	1052.3	1011.9	532

Suppose $s(t) = h$ is the height of the ball as a function of time in seconds.

- (a) (3 points) Compute the average velocity of the ball over the time interval $18 \leq t \leq 30$.

$$\text{Average Velocity} = \frac{h(30) - h(18)}{30 - 18} = \frac{1033 - 815.08}{12} = 18.16 \text{ m/s.}$$

- (b) (3 points) Approximate the instantaneous velocity of the ball when $t = 25$ seconds.

Any of the following are acceptable as an estimate:

$$\text{From the left: } s'(25) \approx \frac{868 - 971.25}{-5} = 20.65 \text{ m/s;}$$

$$\text{From the right: } s'(25) \approx \frac{1033 - 971.25}{5} = 12.35 \text{ m/s;}$$

$$\text{Avg of Left and right: } s'(25) \approx \frac{1033 - 868}{10} = \frac{20.65 + 12.35}{2} = 16.5 \text{ m/s.}$$

- (c) (3 points) Using the information in the table, approximate $s''(25)$. Be sure to carefully show your work. [Note: s'' is approximately constant for this function.]

We can use the information from part (b) to approximate $s''(25)$:

$$s''(25) \approx \frac{s'(30) - s'(20)}{30 - 20} \approx \frac{12.35 - 20.65}{10} = -0.83 \text{ m/s}^2.$$

- (d) (3 points) If the rate of change from $t = 42$ to $t = 60$ were to remain constant, the ball would reach the surface of the moon at approximately $t = 80$. Use the information from the previous parts to decide if this is an overestimate or an underestimate of the time it takes for the ball to reach the ground. **Explain your answer.**

From part (c), s'' is negative (and we are told that s'' is basically constant over all intervals). Thus, the function is concave down. If we extrapolate from $t = 60$ to the vertical axis, we are assuming that the velocity is constant. Since the function s is concave down, the ball would actually hit the surface of the moon before $t = 80$. Thus, $t = 80$ is an overestimate.