9. Cliff is a stage manager for the Royal Shakespeare Company. He notices that the more people that are in the audience, the more nervous the actors get, and consequently they say their lines very fast and the play is shorter. Suppose $T(n)$ is a function that gives the running time of the play in minutes as a function of the number of people, n, in the audience. Cliff also remembers that the play ran 240 minutes at the last dress rehearsal when no one was in the audience and today they had a crowd of 300 people and the show ran for 2 hours.

(a) (3 points) In the context of this problem what is the practical interpretation of $T(158)$?

The expression $T(158)$ gives the number of minutes that the play will run when 158 people are in the audience.

(b) (3 points) In the context of this problem what is the practical interpretation of $T'(158)$?

The expression $T'(158)$ gives the approximate change in the number of minutes that the play will run when the audience increases from 158 to 159 people. The units of T' are minutes per person.

(c) (4 points) Assuming that running time decreases linearly as the number of people attending increases, write an equation for the running time T in minutes as a function of n number of people attending.

We are given two points, (0,240) and (300, 120). Thus, the slope of the linear function is

$$m = \frac{120 - 240}{300 - 0} = \frac{-120}{300} = -0.4.$$

We are given the vertical intercept of (0,240), so an equation of the linear function is

$$T(n) = -0.4n + 240.$$

(d) (4 points) Now assume instead that the running time is exponentially decreasing as a function of the number of the people in the audience, write an equation for the running time T in minutes as a function of the number of people n.

In an equation of the form $T(n) = T_o a^n$, we are given that $T_o = 240$. Using the other given point, we have

$$120 = 240a^{300}$$

so

$$a = 0.5^{1/300} = 0.9977$$

Thus, if the running time is decreasing exponentially, an equation is

$$T(n) = 240(0.9977)^n.$$