

3. Dr. Charles Keeling began measuring carbon dioxide in the atmosphere on a continuous basis in 1958. At that time, Dr. Keeling found that the mean concentration level was approximately 315 ppmv. Currently, the level is approximately 385 ppmv.²

- (a) (4 points) Assuming that the mean concentration has been growing linearly from 1958 to 2007, find a formula for $L(t)$, the mean concentration level of CO₂, with t in years since 1958.

$$L(t) = 315 + \frac{385 - 315}{2007 - 1958} t = 315 + 1.42857 t,$$

where t is the number of years since 1958.

- (b) (5 points) If instead, the mean concentration has been growing at an exponential rate, find an exponential function, $E(t)$, to model the mean concentration level of CO₂ in the environment t years after 1958.

Let t be the number of years from 1958.

$$E(t) = c e^{a t} \text{ at } t = 0, \quad E(0) = 315, \quad \text{therefore } c = 315.$$

Also, $E(49) = 385$, so we have

$$385 = 315 e^{49a}.$$

Taking natural logs:

$$\ln \frac{385}{315} = 49a, \quad \text{therefore } a = \frac{\ln \frac{385}{315}}{49}$$

and

$$E(t) = 315 e^{0.004095 t}.$$

Students may opt to take $E(t) = p b^t$, and $E(0) = 315$ so $p = 315$. Then $E(49) = 385$ gives

$$385 = 315 b^{49}. \quad \text{Rearranging and solving for } b \text{ gives } b = \left(\frac{385}{315}\right)^{1/49},$$

so $E(t) = 315(1.0041)^t$. Note this is equivalent to the answer given above.

- (c) Future CO₂ levels are expected to rise due to burning of fossil fuels and land-use changes. The rate of this increase will depend on uncertain economic, sociological, technological, and natural developments. The IPCC Special Report on Emissions Scenarios gives a wide range of CO₂ scenarios by the year 2100. Use your functions from parts (a) and (b) to predict the concentration of CO₂ in 2100.

1 point taken off if the units are not included.

- (i) (2 points) Prediction if growth is linear:

$$L(142) = 517.857 \text{ ppmv}$$

- (ii) (2 points) Prediction if growth is exponential:

$$E(142) = 563.467 \text{ ppmv}$$

²See http://earthguide.ucsd.edu/globalchange/keeling_curve/01.html and http://en.wikipedia.org/wiki/Carbon_dioxide