

4. The speed of sound,  $v(T)$  (in miles per hour), at an ambient temperature,  $T$  (in degrees Fahrenheit), is given by:

$$v(T) = 740 + 0.4T.$$

Objects which travel faster than the speed of sound create *sonic booms*. However, the ambient temperature  $T$  in the Troposphere also decreases with height  $h$  (in miles) from Earth's surface according to the equation

$$T(h) = -26h + T_0,$$

where  $T_0$  is the temperature at the surface.

- (a) (3 points) Find a formula which will give the speed of sound  $S$  as a function of height  $h$ , assuming the surface temperature is  $68^\circ\text{F}$ .

- (b) (4 points) Find  $S'(1)$  and interpret the meaning of  $S'(1)$  in the context of this problem.

- (c) (3 points) While on a flight from Ann Arbor to Chicago on a beautiful  $68^\circ$  day, the pilot's instruments measure the outside temperature to be  $0^\circ$ . What is the plane's altitude, and how fast would the pilot need to fly at this altitude to create a sonic boom?