1. In 1999, a population of deer (a type of large animal) was set free on a previously uninhabited island in Lake Superior, in an attempt to establish a permanent population of deer on the island. The population of deer grew over time. Population measurements were made each year, as shown in the following table:

| Year | 1999 | 2000 | 2001 | 2002 |
|---|---|---|---|---|
| Population | 20 | 23 | 27 | 31 |

Let $P(t)$ be a function that gives the population of deer on the island as a function of time, $t$, measured in years since 1999.

(a) (2 points) In the context of this problem, give a practical interpretation for $P(40)$.

$P(40)$ represents the deer population on the island in the year 2039.

(b) (2 points) In the context of this problem, give a practical interpretation for $P^{-1}(40)$.

$P^{-1}(40)$ is the number of years after 1999 for which the deer population on the island was 40.

(c) (3 points) Assume that the deer population at time $t$ is represented by an exponential function $P(t) = P_0a^t$. Find $P_0$ and $a$, and express your answer as a function.

From the table we know that at $t = 0$ the population is 20, and also that at $t = 1$ the population is 23. Thus, using the point (0,20) gives the initial value $P_0 = 20$, which gives the equation as $P(t) = 20a^t$. Now, using the second point, (1,23), we can solve for $a$ via $23 = 20a$. Using this, we arrive at the final equation, $P(t) = 20(1.15)^t$.

(d) (2 points) According to your answer to part (c) what is the annual percent growth rate of the deer population?

The annual percent growth rate is the growth factor minus one, expressed in percent form, $(1.15 - 1) \times 100 = 15\%$.

(e) (3 points) Use the table above to estimate $(P^{-1})'(27)$. Do not assume that the deer population is modeled by the formula from part (c).

First, we need some values for $P^{-1}$. From the table we see that $P^{-1}(27) = 2$ and $P^{-1}(31) = 3$. Thus, we can estimate the derivative as $(P^{-1})'(27) \approx \frac{3-2}{31-27} = 0.25$.

(f) (2 points) Give a practical interpretation in the context of this problem for $(P^{-1})'(27)$.

When there were 27 deer on the island, it took approximately 1/4 of a year, or 3 months, for the population to reach 28.