4. The speed of sound, \( v(T) \) (in miles per hour), at an ambient temperature, \( T \) (in degrees Fahrenheit), is given by:

\[
v(T) = 740 + 0.4T.
\]

Objects which travel faster than the speed of sound create **sonic booms**. However, the ambient temperature \( T \) in the Troposphere also decreases with height \( h \) (in miles) from Earth’s surface according to the equation

\[
T(h) = -26h + T_0,
\]

where \( T_0 \) is the temperature at the surface.

(a) (3 points) Find a formula which will give the speed of sound \( S \) as a function of height \( h \), assuming the surface temperature is 68°F.

We are looking for the composite function \( S(h) = v(T(h)) = 740 + 0.4(-26h + 68) = 767.2 - 10.4h \).

(b) (4 points) Find \( S'(1) \) and interpret the meaning of \( S'(1) \) in the context of this problem.

Since \( S \) is linear, the derivative at any point is the same as the line’s slope. Thus, \( S'(1) = -10.4 \frac{mi}{hr} \). Moreover, we can interpret this as telling us that the speed of sound 2 miles above the Earth’s surface is approximately 10.4 mi/hr less than the speed of sound 1 mile above the Earth’s surface.

(c) (3 points) While on a flight from Ann Arbor to Chicago on a beautiful 68°F day, the pilot’s instruments measure the outside temperature to be 0°F. What is the plane’s altitude, and how fast would the pilot need to fly at this altitude to create a sonic boom?

We first need to solve for the plane’s altitude. We can do this by solving the equation \( 0 = -26h + 68 \), which gives \( h \approx 2.61 \) miles. Then, we can find the speed of sound at this altitude via \( S(2.61) \approx 740 \) mi/hr. Thus, at this altitude and ambient temperature the pilot would need to fly faster than this speed in order to create a sonic boom.