4. The speed of sound, $v(T)$ (in miles per hour), at an ambient temperature, T (in degrees Farenheit), is given by:

$$v(T) = 740 + 0.4T.$$

Objects which travel faster than the speed of sound create *sonic booms*. However, the ambient temperature T in the Troposphere also decreases with height h (in miles) from Earth's surface according to the equation

$$T(h) = -26h + T_0,$$

where T_0 is the temperature at the surface.

(a) (3 points) Find a formula which will give the speed of sound S as a function of height h, assuming the surface temperature is $68°$F.

We are looking for the composite function $S(h) = v(T(h)) = 740 + 0.4(-26h + 68) = 767.2 - 10.4h$.

(b) (4 points) Find $S'(1)$ and interpret the meaning of $S'(1)$ in the context of this problem.

Since S is linear, the derivative at any point is the same as the line's slope. Thus, $S'(1) = -10.4\frac{mi/hr}{mi}$. Moreover, we can interpret this as telling us that the speed of sound 2 miles above the Earth's surface is approximately 10.4 mi/hr less than the speed of sound 1 mile above the Earth's surface.

(c) (3 points) While on a flight from Ann Arbor to Chicago on a beautiful $68°$ day, the pilot's instruments measure the outside temperature to be $0°$. What is the plane's altitude, and how fast would the pilot need to fly at this altitude to create a sonic boom?

We first need to solve for the plane's altitude. We can do this by solving the equation $0 = -26h + 68$, which gives $h \approx 2.61$ miles. Then, we can find the speed of sound at this altitude via $S(2.61) \approx 740$ mi/hr. Thus, at this altitude and ambient temperature the pilot would need to fly faster than this speed in order to create a sonic boom.