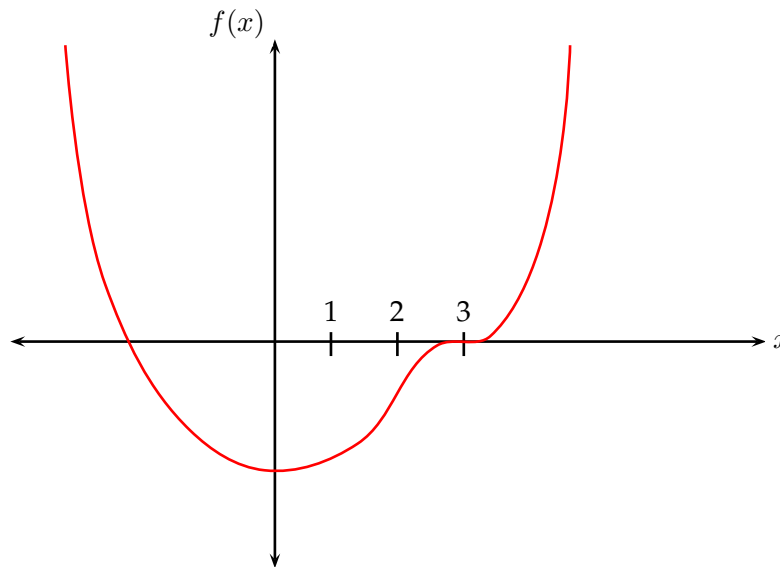7. (9 points) A continuous, differentiable function defined for all x has all of the following properties:

- $f'(x) = 0$ at $x = 0$ and $x = 3$

- $f(3) = 0$

- $f'(-1) = -2$

- f' is increasing for $x < 2$

- $f' \geq 0$ for $x > 0$

- $\lim\limits_{x \to -\infty} f(x) = \infty$

(a) (3 points) Sketch a possible graph of f. (One possible graph is shown below.)

(b) (2 points) How many zeroes does f have? Explain your reasoning.

red f will have infinitely many zeroes if $f' = 0$ for $x \geq 3$, and 2 otherwise. (Either answer is acceptable.)

(c) (2 points) What can you say about the location of the zeroes? Explain your reasoning.

From the given data, we can see that there is one zero for $x < 0$, one zero at $x = 3$, and if $f' = 0$ for $x \geq 3$ we have infinitely many zeroes.

(d) (2 points) Is it possible that $f'(-2) = -1$? Explain your reasoning.

No, since f' is increasing (or equivalently, f is concave up), $f'(-2) < f'(-1)$–(i.e., $f'(-2)$ is more negative).