2. [12 points] Suppose that the line tangent to the graph of $f(x)$ at $x = 3$ passes through the points $(1, 2)$ and $(5, -4)$.

a. [3 points] Find $f'(3)$.

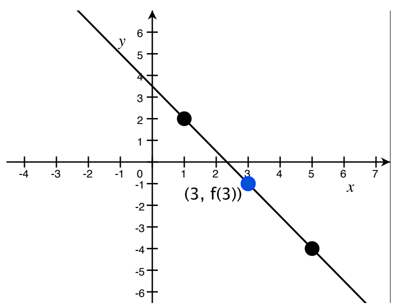
Solution:
The slope of the given tangent line is $\frac{\Delta y}{\Delta x} = \frac{-4-2}{5-1} = \frac{-3}{2}$.
Since $f'(3)$ IS the slope of the tangent line to the graph of $f(x)$ at $x = 3$, we find that $f'(3) = \frac{-3}{2}$.

b. [3 points] Find $f(3)$.

Solution:
$f(3)$ is the y–value of the point on the given line that has x–coordinate equal to 3. Using the slope found in part (a), we find that an equation for the line is

$$y = -\frac{3}{2}x + \frac{7}{2}.$$

So when $x = 3$, it follows that $y = -1$.
Hence $f(3) = -1$. (See graph to right.)



c. [3 points] Estimate the value of $f(2.9)$.

Solution: $f(2.9)$ is approximately equal to the y–value on the given line at the point $x = 2.9$. Using the equation from (b), we find $f(2.9) \approx \left(-\frac{3}{2}\right)(2.9) + \frac{7}{2} = -0.85$. Hence $f(2.9) \approx -0.85$.

d. [3 points] If the graph of f is concave up, is your estimate in part (c) an overestimate, an underestimate, or can you not tell? Explain or demonstrate your answer graphically.

Solution: Since the graph of f is concave up, its slope is increasing. So, the slope of the graph of f between $x = 2.9$ and $x = 3$ is less than (more negative) than $f'(3)$. The point on the graph of f at $x = 2.9$ will therefore be above the point on the line at $x = 2.9$. That is, $f(2.9)$ is greater than the estimate of -0.85 from part (c), so -0.85 is an UNDERESTIMATE of $f(2.9)$. (See graph to right.)