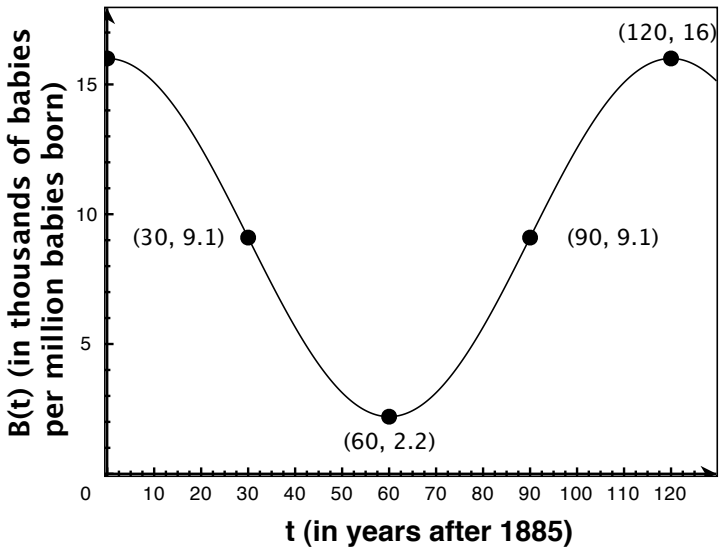
3. [12 points] The popularity of baby names varies over time; the names that are popular one year may not be popular at all within a few years. The popularity of baby names beginning with the letter I appears to be periodic. In 1885, approximately 16,000 per million babies born had first names beginning with the letter I. Their popularity began decreasing at that time and decreased until 1945, when the number had dropped to a low of 2,200 per million. In 2005 it was back to 16,000 per million babies born.
Let $B(t)$ denote the popularity of names beginning with the letter I, in thousands of babies per million babies born, t years after 1885. Assume that $B(t)$ is a sinusoidal function.

a. [6 points] Sketch the graph of $B(t)$. (Remember to clearly label your graph.)

Solution:



b. [6 points] Find a formula for $B(t)$.

Solution: The amplitude of the function is 6.9, period is 120 years (so $B = \frac{2\pi}{120} = \frac{\pi}{60}$) and the midline is at $y = B(t) = 9.1$. Since $t = 0$ corresponds to a maximum, a possible formula for $B(t)$ is

$$B(t) = 6.9\cos\left(\frac{\pi}{60}t\right) + 9.1$$