4. [15 points] The H1N1 flu virus arrived in the US last spring. Data from the CDC for the region including Michigan, Minnesota, Illinois, Indiana, Wisconsin, and Ohio is shown in the table below. $H(t)$ denotes the cumulative number of cases of H1N1 flu in this region $t$ weeks after August 15, 2009.

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H(t)$</td>
<td>8266</td>
<td>8314</td>
<td>8365</td>
<td>8482</td>
<td>8632</td>
<td>8903</td>
<td>9165</td>
</tr>
</tbody>
</table>

a. [2 points] Evaluate and interpret $H(5)$.

Solution: $H(5) = 8903$

This means that 5 weeks after August 15, 2009 (i.e., approximately the third week of September), the cumulative number of cases of H1N1 flu in the midwest region was 8903.

b. [2 points] Why might it be reasonable to assume that $H$ is invertible for $0 \leq t \leq 6$?

Solution: Since $H$ is the cumulative number of cases, it is reasonable that the function is increasing with time over this interval. Thus, we could assume that $H$ is invertible for $0 \leq t \leq 6$.

c. [3 points] Assuming $H$ is invertible, give the practical meaning of $H^{-1}(8500)$.

Solution: The expression $H^{-1}(8500)$ gives us the number of weeks after August 15, 2009 in which the number of cases of H1N1 in the midwest region reached a cumulative total of 8500.

d. [3 points] Estimate $H'(5)$.

Solution: We can approximate $H'(5)$ either by taking

$$\frac{H(5) - H(4)}{5 - 4} = \frac{8903 - 8632}{5 - 4} = 271,$$

or

$$\frac{H(6) - H(5)}{6 - 5} = \frac{9165 - 8903}{6 - 5} = 262,$$

or, we could take the average of the two. Any of the answers are acceptable. The units of the answer are cases per week.

e. [5 points] Assuming $H$ is invertible, estimate and give the practical meaning of $(H^{-1})'(8500)$.

Solution: We can approximate $(H^{-1})'(8500)$ by

$$\frac{(H^{-1})'(8500) \approx \frac{H^{-1}(8632) - H^{-1}(8482)}{8632 - 8482} = \frac{4 - 3}{8632 - 8482} = \frac{1}{150} = 0.0066667 \text{ weeks per case.}$$

The practical interpretation here is that once we had 8500 cases in the region, the next case is likely to be reported in a little over an hour (or for the cumulative number to grow from 8500 to 8501, it would take approximately 1 hour and 7 minutes.)