5. [16 points] Since it was first introduced, the number of users of the internet worldwide has increased dramatically. Let $I(t)$ denote the number (in millions) of worldwide internet users t years after 1995. Then $I(t)$ is given by the formula

$$I(t) = \begin{cases} 16(361/16)^{t/5} & \text{if } 0 \le t \le 5 \\ 361(1.18)^{t-5} & \text{if } 5 < t \le 10 \\ A + 10(t-10) & \text{if } t > 10 \end{cases}$$

a. [3 points] Find A so that $I(t)$ is continuous.

Solution: We want to solve for A so that

$$A + 10(t-10) = 361(1.18)^{t-5} \qquad \text{when } t = 10,$$

so $A = 361(1.18)^5$.

b. [4 points] Find the continuous growth rate of $I(t)$ in the year 1997.

Solution: 1997 corresponds to $t = 2$, so the continuous growth rate is determined by the first piece of the given formula. The annual growth factor is $\frac{361}{16}^{1/5}$ (i.e. the annual growth rate is $\frac{361}{16}^{1/5} - 1$), so we need to find k so that $e^k = \dfrac{361}{16}^{1/5}$, or $k = \ln \dfrac{361}{16}^{1/5} \approx 0.62326$. Thus, the number of users was growing at a continuous rate of approximately 62% in 1997.

c. [3 points] Find the average rate of change of the number of internet users between 1995 and 2000.

Solution: Using the first definition above, we see that there were 16 million users in 1995 and 361 million users in 2000. Thus, the average rate of change for the period is $\dfrac{361-16}{5} = 69$ million users per year.

d. [6 points] Use the definition of the derivative to numerically estimate (i) $I'(7)$ and (ii) $I'(10)$.

Solution: (i) To approximate $I'(7)$, we use $\lim_{h\to 0} \dfrac{361(1.18)^{2+h} - 361(1.18)^2}{h}$, which for small values of h from the left and the right of zero gives approximately 83.19 (to two decimal places). (ii) To approximate $I'(10)$, we must use two limits, first $\lim_{h\to 0^-} \dfrac{361(1.18)^{5+h} - 361(1.18)^5}{h} \approx 136.695$, and second $\lim_{h\to 0^+} \dfrac{A + 10(h) - A}{h} = 10$. Since the limit from the left of $t = 10$ is not equal to the limit from the right of $t = 10$, the limit does not exist and the function is not differentiable at $t = 10$.