3. [10 points] Jim's new car came with an information sheet about the typical fuel efficiency of the car at different speeds. The fuel efficiency, E, is measured in miles per gallon (mpg) and the speed, v, is measured in miles per hour (mph). A portion of the spreadsheet is given here:

E	15	20	22.925	25	26.61	27.925
v	10	20	30	40	50	60

a. [4 points] Jim notices that, for the range of values in this table, v grows exponentially with E. Find an exponential function f so that $v = f(E)$.

Solution: Since f is exponential, it is of the form $f(E) = v_0 a^E$, where v_0 and a are constants.
Since $f(15) = 10$, we have $10 = v_0 a^{15}$. Since $f(20) = 20$, we have $20 = v_0 a^{20}$.
Solving for v_0 in terms of a in each equation we have $\frac{10}{a^{15}} = v_0 = \frac{20}{a^{20}}$.
This means $10a^{20} = 20a^{15}$.
Solving this gives $a = 2^{\frac{1}{5}}$ and $v_0 = \frac{5}{4}$.
Replacing a and v_0 with the numbers found, we have $f(E) = \frac{5}{4}(2)^{\frac{E}{5}}$.

(NOTE: This problem can also be solved with base e which gives $f(E) = 1.25e^{0.1386E}$.)

b. [3 points] Give a practical interpretation of $f^{-1}(17) = 19$.

Solution: The expression $f^{-1}(17) = 19$ means "When Jim's car is going 17 miles per hour, the typical gas mileage of the car is 19 miles per gallon."

c. [3 points] Give a practical interpretation of $(f^{-1})'(25) = 0.3$.

Solution: The expression $(f^{-1})'(25) = 0.3$ means "Jim's car gets about 0.3 miles per gallon less when its speed is 25 miles per hour than when its speed is 26 miles per hour."