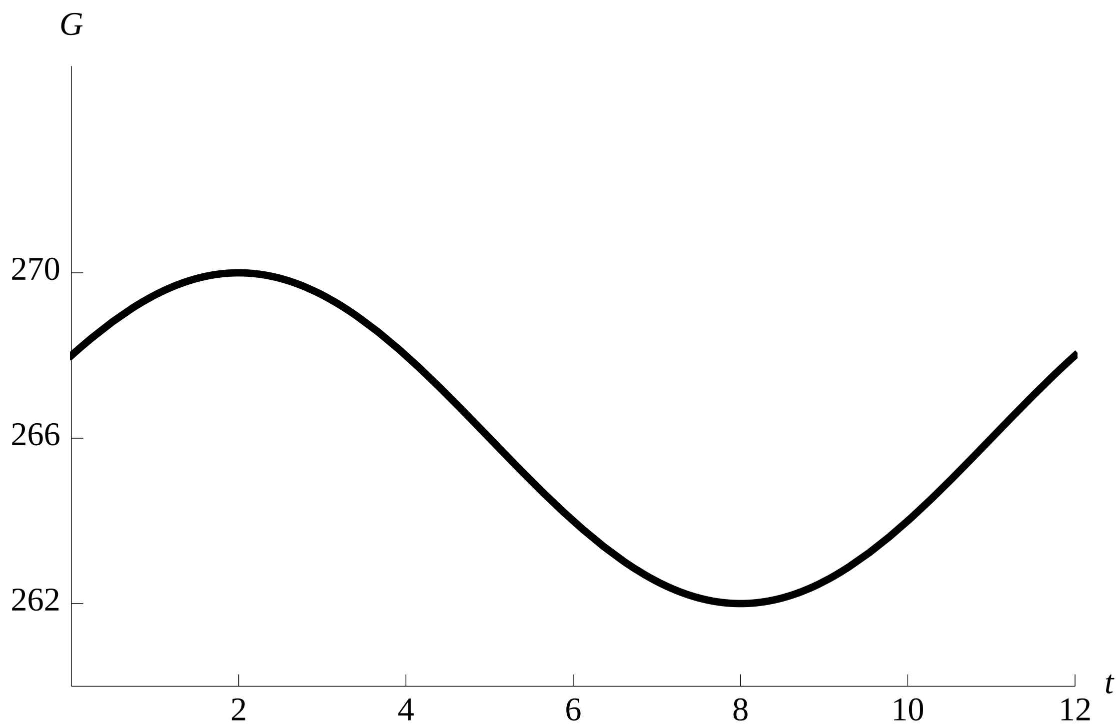
**4**. [10 points] Before the industrial era, the carbon dioxide ($CO_2$) level in the air in Ann Arbor was relatively stable with small seasonal fluctuations caused by plants absorbing $CO_2$ and producing oxygen in its place. Typically, on March 1, the $CO_2$ concentration reached a high of 270 parts per million (ppm), and on September 1, the concentration was at a low of 262 ppm. Let $G(t)$ be the $CO_2$ level $t$ months after January 1.

**a**. [5 points] Assuming that $G(t)$ is periodic and sinusoidal, sketch a neat, *well-labeled* graph of $G$ with $t = 0$ corresponding to January 1.




**b**. [5 points] Determine an explicit expression for $G$, corresponding to your sinusoidal graph above.

*Solution:* The function $G$, being a periodic, sinusiodal function, can be written in the form $G(t) = A\cos(B(t-h)) + k$. Here $A$ is amplitude, $B$ is $2\pi/(\textit{period})$, $h$ is the horizontal shift, and $k$ is the coordinate of the midline. The high point of the graph is on March 1 which corresponds to $t = 2$, so our horizontal shift will be two units to the right meaning $h = 2$. The midline is half way between the high and low values, so $k = (270 + 262)/2 = 266$. The period is 12, so $B = 2\pi/12 = \pi/6$. The amplitude is half of the difference between the high and low values, so $A = (270 - 262)/2 = 4$.
Putting all the pieces together we have

$$G(t) = 4\cos(\frac{\pi}{6}(t-2)) + 266.$$