5. [10 points] Electric cars need large amounts of energy to operate. Most types of batteries, including those found in electric cars, have reduced capacities when discharged at higher rates. For the lithium-ion batteries used in the newest electric cars, this relationship can be expressed by the equation \( C = f(I) = \frac{K}{I^n} \) where \( C \) is the working capacity of the battery in amp hours (Ah) given a discharge rate of \( I \) (with \( n > 1 \)) measured in amps (A). The constant \( K > 0 \) is the rated capacity of the battery.

a. [5 points] Write a formula for the derivative of \( C \) at \( I = 3 \) using the limit definition of the derivative. You do not need to evaluate or simplify this expression.

\[
\frac{dC}{dI}(3) = \lim_{h \to 0} \frac{f(3 + h) - f(3)}{h} = \lim_{h \to 0} \frac{K/(3 + h)^n - K/3^n}{h}.
\]

b. [3 points] Is \( C \) increasing or decreasing at \( I = 3 \)? Justify your answer.

\[
\text{The capacity } C \text{ is decreasing at } I = 3. \text{ To see this, look at the expression for the slope of the secant line on } C \text{ between } I = 3 \text{ and } I = 3 + h \text{ for small } h:
\]
\[
\frac{K/(3 + h)^n - K/3^n}{h}.
\]

The expression is negative if \( h > 0 \) because \( K/(3 + h)^n < K/3^n \). The expression is also negative if \( h < 0 \) because \( K/(3 + h)^n > K/3^n \). Since all of the secant lines near \( I = 3 \) have negative slope, the derivative, which is a limit of the slopes of secant lines, must also be negative.

c. [2 points] What is the concavity of the graph of \( C \) at \( I = 3 \)? Justify your answer.

\[
\text{The concavity of the graph of } C \text{ at } I = 3 \text{ is concave up. As stated in the problem, the capacity } C \text{ is of the form } C = \frac{K}{I^n}. \text{ This function is a scalar multiple of the power function } I^{-n} \text{ which is concave up for } I > 0 \text{ as long as } n > 0.
\]

Another approach is to use the power rule to differentiate the function twice. Starting with \( C = f(I) = KI^{-n} \), we have
\[
f'(I) = -nKI^{-n-1}
\]
and
\[
f''(I) = (-n)(-n-1)KI^{-n-2} = n(n+1)KI^{-n-2}.
\]
Now for \( I > 0 \), \( f''(I) > 0 \) meaning \( C = f(I) \) is concave up.