1. [12 points] For each part below, give an explicit formula for a function which satisfies the given properties, if one exists. If such a function does not exist, explain why. Be sure to clearly indicate your final answer for each part.

a. [3 points] A continuous function, \( f \), which is not differentiable.

\[\text{Solution:}\] The function \( f(x) = |x| \) is continuous, but not differentiable. The function is continuous as it can be drawn without picking up one’s pencil, but not differentiable because there is a corner on the graph at the point \((0, 0)\).

b. [3 points] A cubic polynomial, \( p \), with two \( x \)-intercepts.

\[\text{Solution:}\] The function \( p(x) = x^2(x - 1) = x^3 - x^2 \) is a cubic polynomial with \( x \)-intercepts at \( x = 0, 1 \).

c. [3 points] A continuous function, \( c \), satisfying \( \lim_{x \to 0^+} c(x) = -1 \) and \( \lim_{x \to 0^-} c(x) = 1 \).

\[\text{Solution:}\] The function described here does not exist. If \( \lim_{x \to 0^+} c(x) = -1 \) and \( \lim_{x \to 0^-} c(x) = 1 \), then \( \lim_{x \to 0} c(x) \) does not exist since the right and left hand limits are not equal. The function \( c(x) \) is continuous at zero means the limit as \( x \to 0 \) exists and equals \( c(0) \). If the right hand limit and the left hand limit are not the same, \( \lim_{x \to 0} c(x) \) does not exist, and so \( c(x) \) cannot be continuous at \( x = 0 \).

d. [3 points] A rational function, \( r \), with a vertical asymptote at \( x = 1 \) and a horizontal asymptote at \( y = 1 \).

\[\text{Solution:}\] The function \( r(x) = \frac{x}{x - 1} \) has a vertical asymptote at \( x = 1 \) and a horizontal asymptote at \( y = 1 \).