1. [12 points] For each part below, give an explicit formula for a function which satisfies the given properties, if one exists. If such a function does not exist, explain why. Be sure to clearly indicate your final answer for each part.

a. [3 points] A continuous function, f, which is not differentiable.

Solution: The function $f(x) = |x|$ is continuous, but not differentiable. The function is continuous as it can be drawn without picking up one's pencil, but not differentiable because there is a corner on the graph at the point $(0, 0)$.

b. [3 points] A cubic polynomial, p, with two x-intercepts.

Solution: The function $p(x) = x^2(x-1) = x^3 - x^2$ is a cubic polynomial with x-intercepts at $x = 0, 1$.

c. [3 points] A continuous function, c, satisfying $\lim_{x\to 0^+} c(x) = -1$ and $\lim_{x\to 0^-} c(x) = 1$.

Solution: The function described here does not exist. If $\lim_{x\to 0^+} c(x) = -1$ and $\lim_{x\to 0^-} c(x) = 1$, then $\lim_{x\to 0} c(x)$ does not exist since the right and left hand limits are not equal. The function $c(x)$ is continuous at zero means the limit as $x \to 0$ exists and equals $c(0)$. If the right hand limit and the left hand limit are not the same, $\lim_{x\to 0} c(x)$ does not exist, and so $c(x)$ cannot be continuous at $x = 0$.

d. [3 points] A rational function, r, with a vertical asymptote at $x = 1$ and a horizontal asymptote at $y = 1$.

Solution: The function $r(x) = \frac{x}{x-1}$ has a vertical asymptote at $x = 1$ and a horizontal asymptote at $y = 1$.