

2. [12 points] The Facebook Data team has decided to track the University of Michigan network status updates that mention football in order to see which days to show ads for tailgating supplies. Starting at 1pm Saturday, they measure an aggregate Football Status Factor by calculating the percentage of status updates which mention any of a number of designated football terms every hour. They notice very quickly that the data is sinusoidal with period 168 hours (the number of hours in a week). Suppose $F(t)$ is this percentage, t hours after 1pm Saturday.

a. [2 points] If the maximum percentage is 96% at 1pm Saturday, and the minimum is 28% attained 84 hours later, compute the following quantities:

1. Midline

Solution: Since the maximum value is 96 and the minimum value is 28, the midline will be the average of these two values, that is $(96 + 28)/2 = 62$.

2. Amplitude

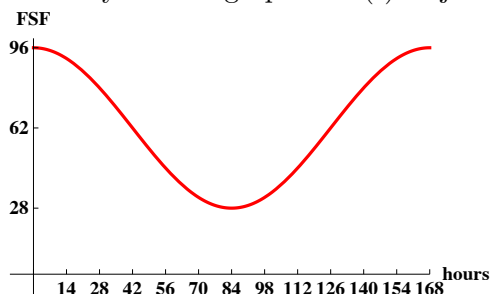
Solution: The amplitude of this sinusoidal function is half the distance between the maximum and minimum values, that is $(96 - 28)/2 = 34$. Notice that the midline plus the amplitude will be the maximum $62 + 34 = 96$ and the midline minus the amplitude will be the minimum $62 - 34 = 28$.

b. [6 points] Using the values computed above, find a formula for $F(t)$.

Solution: From the problem, we know that this function begins at its maximum, which makes cosine the natural choice for our sinusoidal function. We need the appropriate values of A, B , and k in $F(t) = A \cos(Bt) + k$. We are given that the period is 168 hours, so the value of $B = 2\pi/168$. We found the amplitude $A = 34$ and the midline $k = 62$ in part (a). So, the formula for $F(t)$ will be

$$F(t) = 34 \cos\left(\frac{2\pi t}{168}\right) + 62.$$

c. [4 points] Suppose advertisers want to advertise when the rate at which people are talking about football is increasing the fastest. What time range would you recommend to them and why? Use a graph of $F(t)$ to justify your answer.



Solution: The Football Status factor gives an hourly rate of people talking about football since it averages mentions of football in Facebook statuses over the course of an hour. To find when the rate at which people are talking about football is increasing the fastest, we just need to find when the Football Status Factor or $F(t)$ is increasing the fastest. The function F' will tell us whether $F(t)$ is increasing or decreasing, but to find maximum of F' , we need to consider F'' , the second derivative of F . At $t = 126$ hours, the graph is increasing and switches from concave up to concave down, so the derivative is at its largest when $t = 126$. Any time range around $t = 126$ would be the best time for advertisers to advertise.