5. [6 points] Find a number k so that the following function is continuous on any interval.

$$j(t) = \begin{cases} (t+4)^3 & t < -2 \\ kt & t \geq -2 \end{cases}$$

Using your value of k, explain why this function is continuous on any interval.

Solution: On intervals not containing $t = -2$, this function is continuous since the functions $(t+4)^3$ and kt are both polynomials, and thus continuous, regardless of the value of k. So we must find the value of k which makes $j(t)$ continuous at $t = -2$. So we set

$$\lim_{t \to -2^-} j(t) = (-2+4)^3 = -2k = \lim_{t \to -2^+} j(t).$$

Solving this, we get $k = -4$. Now for any interval containing $t = -2$ we have that $j(t)$ is continuous.

6. [5 points] Using the limit definition of the derivative, write an explicit expression for the derivative of the function $E(x) = x^{\cos x}$ at $x = 2$. Do not try to calculate this derivative.

Solution:

$$E'(2) = \lim_{h \to 0} \frac{(2+h)^{\cos(2+h)} - 2^{\cos 2}}{h}.$$