4. [12 points] The Dow Jones Industrial Average (DJIA) is a stock market index which measures how the stocks of 30 large publicly-owned companies perform during a given period of time. On September 27, 2012 at 11:30am the DJIA was 13,420 and at 1:30pm, the DJIA was 13,520. Suppose $A = h(t)$ gives the value of the DJIA t hours after 9:00am on September 27, 2012 with $0 \leq t \leq 8$.

a. [4 points] Using the information given above approximate A using a linear function, $\ell(t)$. Write an expression for $\ell(t)$.

Solution: We are given $\ell(2.5) = 13420$ and $\ell(4.5) = 13520$. So the slope is $\dfrac{13,520 - 13,420}{4.5 - 2.5} = \dfrac{100}{2} = 50$. Now we can find our vertical intercept, b, by plugging in a point and solving: $13,420 = 50 \cdot 2.5 + b$ so $b = 13,295$.

$\ell(t) =$ $50t + 13,295$

b. [4 points] Your friend tells you that an exponential function would be more accurate in modeling A. If $g(t)$ is an exponential function which approximates A, what is the hourly growth rate of $g(t)$? What was the value of the DJIA at 2:30pm on September 27, 2012 according to this model?

Solution: 11:30 and 1:30 are two hours apart, so if a is the growth factor, $13420a^2 = 13520$, so $a \approx 1.0037$. So the growth rate is 0.0037, or 0.37%.
By 2:30, the DJIA will grow by one more growth factor, so it will be at $(13520)(1.0037) \approx 13570.3$

growth rate = 0.37%   value of DJIA at 2:30pm = 13570.3

c. [4 points] In the end you realize the best model for A is a function of the form

$$p(t) = t^k + b$$

where k and b are constants. You also find out that at 9am on September 27, the DJIA was actually 13,402. Find values of k and b so that $p(t)$ approximates A.

Solution: We are given $p(0) = 13402$, so $b = 13402$.
We can use either of the other two points to get a value for k. Using the point at $t = 2.5$, we get

$$2.5^k + 13402 = 13420.$$

$$2.5^k = 18.$$

$$k \ln 2.5 = \ln 18.$$

This gives $k \approx 3.154$ (if we used the other point, we would get $k \approx 3.172$).

$k =$ 3.154 $b =$ 13,402