7. [13 points] f is a continuous, differentiable function defined for all real numbers. Some values of f and its derivative are given in the table below.

x	0	1	2	3	4	5	6	7
$f(x)$	-11.2	-4.0	-1.1	-0.5	-0.1	2.0	7.9	19.6
$f'(x)$	9.9	4.7	1.4	0.2	0.9	2.1	5.9	11.7

a. [4 points] Estimate the derivative of f at $x = 5, 6$, and 7, and fill in the remainder of the table.

Solution: Using left-hand estimates,
$f'(5) \approx \dfrac{2.0 - (-0.1)}{5 - 4} = 2.1$
$f'(6) \approx 7.9 - 2.0 = 5.9$
$f'(7) \approx 19.6 - 7.9 = 11.7.$

b. [2 points] Estimate $f''(1)$ using the data given.

Solution: We can use a left-hand estimate, a right-hand estimate, or find both and average them:
Left: $4.7 - 9.9 = -5.2$
Right: $1.4 - 4.7 = -3.3$
Average: $\dfrac{-5.2 - 3.3}{2} = -4.25$

c. [4 points] Assuming the concavity of f doesn't change on the interval $5 \leq x \leq 7$, is the graph of f concave up or concave down on that interval? Explain.

Solution: f is concave up on the interval $5 \leq x \leq 7$, because our estimates for the derivative are increasing on this interval.

d. [3 points] Using your answer from part (c), is your approximation for $f'(7)$ an overestimate or an underestimate? Explain.

Solution: Our approximation for $f'(7)$ is an underestimate. Our estimate was the slope of a secant line on the left, which will be smaller than the slope of the tangent line since f is concave up.