7. [13 points] \( f \) is a continuous, differentiable function defined for all real numbers. Some values of \( f \) and its derivative are given in the table below.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>-11.2</td>
<td>-4.0</td>
<td>-1.1</td>
<td>-0.5</td>
<td>-0.1</td>
<td>2.0</td>
<td>7.9</td>
<td>19.6</td>
</tr>
<tr>
<td>( f'(x) )</td>
<td>9.9</td>
<td>4.7</td>
<td>1.4</td>
<td>0.2</td>
<td>0.9</td>
<td>2.1</td>
<td>5.9</td>
<td>11.7</td>
</tr>
</tbody>
</table>

**a.** [4 points] Estimate the derivative of \( f \) at \( x = 5, 6, \) and 7, and fill in the remainder of the table.

\[
\begin{align*}
  f'(5) & \approx \frac{2.0 - (-0.1)}{5 - 4} = 2.1 \\
  f'(6) & \approx 7.9 - 2.0 = 5.9 \\
  f'(7) & \approx 19.6 - 7.9 = 11.7.
\end{align*}
\]

**b.** [2 points] Estimate \( f''(1) \) using the data given.

\[
\begin{align*}
  &\text{Solution: We can use a left-hand estimate, a right-hand estimate, or find both and average them:} \\
  &\text{Left: } 4.7 - 9.9 = -5.2 \\
  &\text{Right: } 1.4 - 4.7 = -3.3 \\
  &\text{Average: } \frac{-5.2 - 3.3}{2} = -4.25
\end{align*}
\]

**c.** [4 points] Assuming the concavity of \( f \) doesn’t change on the interval \( 5 \leq x \leq 7 \), is the graph of \( f \) concave up or concave down on that interval? Explain.

\[
\begin{align*}
  &\text{Solution: } f \text{ is concave up on the interval } 5 \leq x \leq 7, \text{ because our estimates for the derivative are increasing on this interval.}
\end{align*}
\]

**d.** [3 points] Using your answer from part (c), is your approximation for \( f'(7) \) an overestimate or an underestimate? Explain.

\[
\begin{align*}
  &\text{Solution: } \text{Our approximation for } f'(7) \text{ is an underestimate. Our estimate was the slope of a secant line on the left, which will be smaller than the slope of the tangent line since } f \text{ is concave up.}
\end{align*}
\]