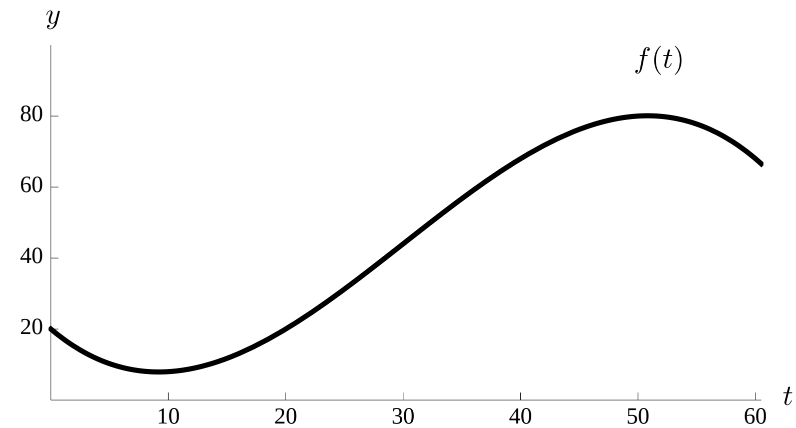
8. [14 points] Your pet bird is flying in a straight path toward you and away from you for a minute. After t seconds, she is $f(t)$ feet away from you, where

$$f(t) = \frac{-t(t-20)(t-70)}{500} + 20, \qquad 0 \le t \le 60.$$

A graph of $y = f(t)$ is shown here.



a. [3 points] Without doing any calculations, determine which is greater: the average velocity of the bird over the entire minute, or her instantaneous velocity after 30 seconds. Explain, referring to the graph.

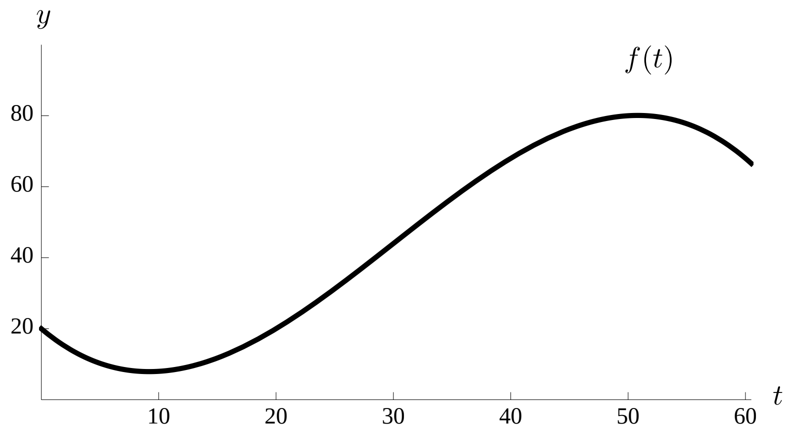
Solution: The slope of the secant line from $t = 0$ to $t = 60$ is the average velocity over the minute and slope of the tangent line at $t = 30$ is the instantaneous velocity after 30 seconds. If you draw these lines on the graph, you can see that the tangent line clearly has a larger slope. Thus, her instantaneous velocity after 30 seconds is greater than her average velocity over the entire minute.

b. [3 points] Calculate the exact value of the average velocity of the bird over the entire minute.

Solution: Average velocity $= \dfrac{f(60) - f(0)}{60 - 0} = \dfrac{68 - 20}{60} = 0.8$ ft/s

8. (continued) The formula for f and its graph are repeated below for your convenience.

$$f(t) = \frac{-t(t-20)(t-70)}{500} + 20, \qquad 0 \le t \le 60.$$



c. [4 points] Write an explicit expression for the velocity of the bird at time t using the limit definition of velocity. Final answers containing the letter f will receive no credit. Do not evaluate your expression.

Solution: $$f'(t) = \lim_{h\to 0} \frac{\left(\frac{-(t+h)(t+h-20)(t+h-70)}{500} + 20\right) - \left(\frac{-t(t-20)(t-70)}{500} + 20\right)}{h}$$

d. [4 points] After a minute, you scare the bird, and she flies away at 9 feet/sec. Write a formula for a continuous function $f(t)$ describing the distance between you and the bird for $0 \le t \le 180$.

Solution: After 60 seconds, the bird is $f(60) = 68$ ft away. So we want to find a formula for a line with slope 9 passing through $(60, 68)$. Plugging in and solving for the vertical intercept b, we get $68 = 9 \cdot 60 + b$. So $b = -472$. We can then write this as a piecewise function:

$$f(t) = \begin{cases} \frac{-t(t-20)(t-70)}{500} + 20 & 0 \le t \le 60 \\ 9t - 472 & 60 < t \le 180 \end{cases}$$