8. [14 points] Your pet bird is flying in a straight path toward you and away from you for a minute. After $t$ seconds, she is $f(t)$ feet away from you, where

$$f(t) = \frac{-t(t - 20)(t - 70)}{500} + 20, \quad 0 \leq t \leq 60.$$ 

A graph of $y = f(t)$ is shown here.

\[ \text{Graph of } y = f(t) \]

a. [3 points] Without doing any calculations, determine which is greater: the average velocity of the bird over the entire minute, or her instantaneous velocity after 30 seconds. Explain, referring to the graph.

Solution: The slope of the secant line from $t = 0$ to $t = 60$ is the average velocity over the minute and slope of the tangent line at $t = 30$ is the instantaneous velocity after 30 seconds. If you draw these lines on the graph, you can see that the tangent line clearly has a larger slope. Thus, her instantaneous velocity after 30 seconds is greater than her average velocity over the entire minute.

b. [3 points] Calculate the exact value of the average velocity of the bird over the entire minute.

Solution: Average velocity $= \frac{f(60) - f(0)}{60 - 0} = \frac{68 - 20}{60} = 0.8$ ft/s
8. (continued) The formula for $f$ and its graph are repeated below for your convenience.

$$f(t) = \frac{-t(t - 20)(t - 70)}{500} + 20, \quad 0 \leq t \leq 60.$$ 

![Graph of $f(t)$]

c. [4 points] Write an explicit expression for the velocity of the bird at time $t$ using the limit definition of velocity. Final answers containing the letter $f$ will receive no credit. Do not evaluate your expression.

$$\text{Solution:} \quad f'(t) = \lim_{h \to 0} \frac{-t+h(t+h-20)(t+h-70)}{500 + 20}$$

d. [4 points] After a minute, you scare the bird, and she flies away at 9 feet/sec. Write a formula for a continuous function $f(t)$ describing the distance between you and the bird for $0 \leq t \leq 180$.

$$\text{Solution:} \quad \text{After 60 seconds, the bird is } f(60) = 68 \text{ ft away. So we want to find a formula for a line with slope 9 passing through (60, 68). Plugging in and solving for the vertical intercept } b, \text{ we get } 68 = 9 \cdot 60 + b. \text{ So } b = -472. \text{ We can then write this as a piecewise function:}$$

$$f(t) = \begin{cases} \frac{-t(t - 20)(t - 70)}{500} + 20 & 0 \leq t \leq 60 \\ 9t - 472 & 60 < t \leq 180 \end{cases}$$