1. [14 points] Carla is trying to model the growth of the feet of her son, Taser, to predict what size boots she needs to buy him to last him through the winter. She has measured Taser's feet three times, once exactly nine months ago, once exactly three months ago, and once just today. Carla decides to measure t in months since she took her first measurement. Below is a table containing her measurements. Carla lost the record of her first measurement so the corresponding entry in the table is blank.

t (months)	0	6	9
foot length (inches)		6.4	7.2

a. [3 points] Write a linear function $L(t)$ modeling the length of Taser's feet t months after she took her first measurement.

Solution: The slope of $L(t)$ is $\dfrac{L(9)-L(6)}{9-6} = \dfrac{7.2-6.4}{9-6} = \dfrac{0.8}{3} \approx 0.267$. Using point-slope form gives

$$L(t) = \frac{0.8}{3}(t-6) + 6.4 = \frac{0.8}{3}t + 4.8$$

b. [5 points] Write a exponential function $E(t)$ modeling the length of Taser's feet t months after she took her first measurement.

Solution: An exponential function $E(t)$ will be of the form $E(t) = ab^t$. Plugging in the data points gives

$$\begin{aligned} 6.4 &= ab^6 \\ 7.2 &= ab^9. \end{aligned}$$

Dividing the second equation by the first gives $b^3 = \dfrac{7.2}{6.4}$ so $b = \left(\dfrac{7.2}{6.4}\right)^{1/3} \approx 1.040$. Using the first equation above, we have

$$6.4 = a\left(\left(\frac{7.2}{6.4}\right)^{1/3}\right)^6$$

so $a = \dfrac{6.4}{\left(\dfrac{7.2}{6.4}\right)^2} \approx 5.057$. Therefore our function is $E(t) = 5.057(1.040)^t$.

c. [2 points] According to the exponential model you found in **(b)**, what is the missing value in the table above?

Solution:

$$E(0) \approx 5.057$$

d. [4 points] Bob, the salesman at the shoe store, has a different model for foot growth. He gives Carla the formula

$$B(t) = \frac{50}{5 + 6e^{-t/8}}$$

for the length of Taser's feet t months since Carla took her first measurement. According to Bob's model, when will Taser's feet be 8 inches long? Give your answer in exact form with no decimals.

Solution: We need to solve $B(t) = 8$ for t.

$$\begin{aligned}
\frac{50}{5 + 6e^{-t/8}} &= 8 \\
\frac{50}{8} &= 5 + 6e^{-t/8} \\
\frac{\frac{50}{8} - 5}{6} &= e^{-t/8} \\
\ln \frac{\frac{50}{8} - 5}{6} &= -t/8 \\
-8 \ln \frac{\frac{50}{8} - 5}{6} &= t
\end{aligned}$$