- 4. [7 points] After the success of his new bacon-flavored soda, Louis wants to try making a flavor that customers will find more refreshing in the hot summer months. Louis notices daily sales of his new spearmint soda vary seasonally. Sales reach a high of \$300 around August 1 and a low of \$120 around February 1. Suppose that daily sales of the soda (in dollars) can be modeled by a sinusoidal function S(t) where t is the time in months since January 1. Note that August 1 is seven months after January 1. You do not need to show work for this problem.
  - **a**. [2 points] What are the period and amplitude of the function S(t)?

Period = \_\_\_\_\_ 12 \_\_\_\_\_
Amplitude = \_\_\_\_\_ 90

**b.** [5 points] Write a formula for the function S(t).

Solution:  

$$S(t) = -90 \cos\left(\frac{2\pi}{12}(t-1)\right) + 210 = 90 \sin\left(\frac{2\pi}{12}(t-4)\right) + 210 = 90 \cos\left(\frac{2\pi}{12}(t-7)\right) + 210$$
(There are other possible answers.)

5. [6 points] For which value(s) of a is the following function continuous? Show all of your work.

$$f(x) = \begin{cases} \frac{x^2 - 9}{x - 3} & \text{for } x < 3\\ ax^2 + 2x + 15 & \text{for } x \ge 3 \end{cases}$$

Solution: The function f(x) is continuous everywhere except possibly at x = 3. First find the left-hand limit at x = 3:

$$\lim_{x \to 3^{-}} f(x) = \lim_{x \to 3^{-}} \frac{x^2 - 9}{x - 3} = \lim_{x \to 3^{-}} \frac{(x + 3)(x - 3)}{x - 3} = \lim_{x \to 3^{-}} x + 3 = 6$$

Then find the right-hand limit at x = 3:

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} ax^2 + 2x + 15 = 9a + 21$$

We need the left and right limits at 3 to be equal. So we solve 6 = 9a + 21 to get  $a = -\frac{5}{3}$ . The function with  $a = -\frac{5}{3}$  is continuous at x = 3 because  $\lim_{x\to 3} f(x) = 6$  and f(3) = 6.