4. [7 points] After the success of his new bacon-flavored soda, Louis wants to try making a flavor that customers will find more refreshing in the hot summer months. Louis notices daily sales of his new spearmint soda vary seasonally. Sales reach a high of \$300 around August 1 and a low of \$120 around February 1. Suppose that daily sales of the soda (in dollars) can be modeled by a sinusoidal function $S(t)$ where t is the time in months since January 1. Note that August 1 is seven months after January 1. You do not need to show work for this problem.

a. [2 points] What are the period and amplitude of the function $S(t)$?

Period = **12**

Amplitude = **90**

b. [5 points] Write a formula for the function $S(t)$.

Solution:

$$S(t) = -90\cos\left(\frac{2\pi}{12}(t-1)\right)+210 = 90\sin\left(\frac{2\pi}{12}(t-4)\right)+210 = 90\cos\left(\frac{2\pi}{12}(t-7)\right)+210$$

(There are other possible answers.)

5. [6 points] For which value(s) of a is the following function continuous? Show all of your work.

$$f(x) = \begin{cases} \dfrac{x^2-9}{x-3} & \text{for } x < 3 \\ ax^2+2x+15 & \text{for } x \geq 3 \end{cases}$$

Solution: The function $f(x)$ is continuous everywhere except possibly at $x = 3$. First find the left-hand limit at $x = 3$:

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{x^2-9}{x-3} = \lim_{x\to 3^-} \frac{(x+3)(x-3)}{x-3} = \lim_{x\to 3^-} x+3 = 6$$

Then find the right-hand limit at $x = 3$:

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} ax^2+2x+15 = 9a+21$$

We need the left and right limits at 3 to be equal. So we solve $6 = 9a+21$ to get $a = -\dfrac{5}{3}$.
The function with $a = -\dfrac{5}{3}$ is continuous at $x = 3$ because $\lim_{x\to 3} f(x) = 6$ and $f(3) = 6$.