4. [7 points] After the success of his new bacon-flavored soda, Louis wants to try making a flavor that customers will find more refreshing in the hot summer months. Louis notices daily sales of his new spearmint soda vary seasonally. Sales reach a high of $300 around August 1 and a low of $120 around February 1. Suppose that daily sales of the soda (in dollars) can be modeled by a sinusoidal function $S(t)$ where $t$ is the time in months since January 1. Note that August 1 is seven months after January 1. You do not need to show work for this problem.

a. [2 points] What are the period and amplitude of the function $S(t)$?

Period = \underline{12}

Amplitude = \underline{90}

b. [5 points] Write a formula for the function $S(t)$.

Solution:

$S(t) = -90 \cos \left( \frac{2\pi}{12} (t - 1) \right) + 210 = 90 \sin \left( \frac{2\pi}{12} (t - 4) \right) + 210 = 90 \cos \left( \frac{2\pi}{12} (t - 7) \right) + 210$

(There are other possible answers.)

5. [6 points] For which value(s) of $a$ is the following function continuous? Show all of your work.

$f(x) = \begin{cases} \frac{x^2 - 9}{x - 3} & \text{for } x < 3 \\ ax^2 + 2x + 15 & \text{for } x \geq 3 \end{cases}$

Solution: The function $f(x)$ is continuous everywhere except possibly at $x = 3$. First find the left-hand limit at $x = 3$:

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} \frac{x^2 - 9}{x - 3} = \lim_{x \to 3^-} \frac{(x + 3)(x - 3)}{x - 3} = \lim_{x \to 3^-} x + 3 = 6$

Then find the right-hand limit at $x = 3$:

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} ax^2 + 2x + 15 = 9a + 21$

We need the left and right limits at 3 to be equal. So we solve $6 = 9a + 21$ to get $a = -\frac{5}{3}$.

The function with $a = -\frac{5}{3}$ is continuous at $x = 3$ because $\lim_{x \to 3} f(x) = 6$ and $f(3) = 6$. 

University of Michigan Department of Mathematics

Fall, 2013 Math 115 Exam 1 Problem 5 (Corey’s cats) Solution