

8. [12 points] In Ann Arbor, the average property value  $P$ , in dollars per square foot, can be modeled as a function of the distance  $x$ , in miles, you are away from the city center. This relationship can be written  $P = g(x)$ . Below is a table containing information about  $g(x)$ . Use the information in the table to answer the parts of this question.

$x$	0.1	0.2	0.3	0.4	0.5
$g(x)$	200	162	142	130	119
$g'(x)$	-401	-298	-160	-115	-118

- a. [3 points] Estimate  $g'(0.15)$  using only values of  $g(x)$  from the table.

*Solution:*

$$g'(0.15) \approx \frac{g(0.2) - g(0.1)}{0.2 - 0.1} = \frac{162 - 200}{0.1} = -380$$

- b. [3 points] Estimate  $g''(0.45)$  using only values of  $g'(x)$  from the table.

*Solution:*

$$g''(0.45) \approx \frac{g'(0.5) - g'(0.4)}{0.5 - 0.4} = \frac{-118 - (-115)}{0.1} = -30$$

- c. [3 points] Assuming the concavity of  $g(x)$  does not change on the interval  $0.1 < x < 0.3$ , do you expect  $g(x)$  to be concave up, concave down, or neither over this interval? Explain.

*I expect  $g(x)$  to be **concave up** because ...*

*Solution:* ...  $g'(x)$  is increasing on the interval  $0.1 < x < 0.3$ .

...  $g(x)$  decreases more between  $x = 0.1$  and  $x = 0.2$  than between  $x = 0.2$  and  $x = 0.3$ .

- d. [3 points] Write a sentence expressing the meaning of

$$g'(0.3) = -160$$

which could be understood by someone who knows no calculus. The beginning of the sentence is given below.

*If I am 0.3 miles from the center of Ann Arbor looking at properties and I travel 0.05 miles toward the city center, ...*

*Solution:* ... average property values will increase by about 8 dollars per square foot.